Lining Up: The Relationship between the Common Core State Standards and Five Sets of Comparison Standards

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Executive Summary

In June 2010, the National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO) released the Common Core State Standards.[1] The stated aim of the Common Core standards is to define the knowledge and skills students should achieve in order to graduate from high school ready to succeed in a wide range of postsecondary educational settings (Common Core State Standards Initiative, 2010a).

As of October 2011, 44 states and the District of Columbia had officially adopted the standards. This widespread potential implementation of the Common Core standards has led to interest by states and national organizations regarding the relationship between these new standards and existing systems of standards. Are the Common Core standards aligned with the standards states and others have developed over many years? Are they as challenging? Do they cover the same topic areas with the same emphases?

To help answer these questions, the Educational Policy Improvement Center (EPIC), designed and conducted this study to determine the extent of correspondence (alignment) between the exit level Common Core standards and each of five sets of existing standards. The sets of standards were selected because they were either identified as exemplary state standards, were explicitly written at the college readiness level, or represented a rigorous instructional program focused on college readiness. The purpose was to see if the Common Core standards cover similar content, how broadly they cover the comparison standards, and how the cognitive challenge level of aligned content matches up.

The study asks three questions about the Common Core standards in English language arts and literacy, and mathematics:

Alignment

The Educational Policy Improvement Center designed and conducted this study to determine the extent of correspondence (alignment) between the exit level Common Core State Standards and each of five sets of existing standards. The sets of standards were selected because they were either identified as exemplary state standards, were explicitly written at the college readiness level, or represented a rigorous instructional program focused on college readiness.

[1] Referred to hereafter as the Common Core standards.

1. To what extent are the knowledge and skills found in the comparison standards the same as or different from what is described in the Common Core standards? (match)

2. What is the cognitive complexity level of the Common Core standards and to what extent are the matched comparison standards at a higher or lower level of cognitive complexity? (depth)

3. How broadly do the matched comparison standards cover the content of the Common Core standards? (breadth)

Study Overview

Comparison Standards

The comparison standards selected for the study come from two states that have been regarded as having high quality educational standards: California and Massachusetts (Achieve, Inc., 2010). The Texas College and Career Readiness Standards are included because they represent one of the only sets of competencies and skill statements developed by a postsecondary education agency in collaboration with K-12 educators (Texas Higher Education Coordinating Board & Texas Education Agency, 2008). The other set of college readiness standards, the Knowledge and Skills for University Success (KSUS), were developed by university faculty in the early 2000s and represent the first set of such standards (Conley, 2003). Finally, to capture a more international perspective, the standards from the International Baccalaureate (IB) Diploma Programme are also examined. The IB Diploma Programme is offered in 141 countries and is becoming increasingly popular in the US.

The specific comparison standards are as follows:

- California: The Content Standards for California Public Schools, for the 11th–12th grade band in English language arts and for 8th–12th grade band in mathematics (released in 1997)
- Massachusetts: The Massachusetts Curriculum Frameworks, for the 11th–12th grade band in English language arts (released in 2001) and mathematics (released in 2000)
- Texas: The Texas College and Career Readiness Standards in English/language arts, mathematics, and cross-disciplinary standards (released in 2008)
- KSUS: The Knowledge and Skills for University Success (KSUS) standards in English and mathematics,[2] developed as college-preparatory standards by Standards for Success (released in 2003)
- IB: The International Baccalaureate Diploma Programme English language arts and mathematics standards, developed by EPIC, for IB's Programs of Study for 10th–12th grades[3] (released in 2009)

Methodology

We adopted and adapted Cook and Wilmes's (2007) standards-to-standards alignment methodology that includes a combination of linking (match between standards) and correspondence (depth and breadth). The method derives from Webb's alignment methodology (1997, 1999, 2002a). The Cook and Webb approaches have been widely used to assess alignment of assessments and standards. The method employs panels of experts to review and rate content and cognitive processes for each standard and then use the expert ratings to calculate statistics of alignment. These statistics demonstrate how closely the assessment reflects the standards in terms of content and cognitive challenge (Webb, Herman, & Webb, 2007). Standards-to-standards alignment (Cook, 2007; Cook & Wilmes, 2007) provides a means to quantify and evaluate the extent of overlap between different sets of standards in terms of knowledge, skills, and content.

Three alignment indices provide the data to answer the study's three research questions:

1. **Categorical Concurrence:** the extent of overlap or match between the comparison standards and the Common Core standards.

[2]These did not include the Knowledge and Skills Foundations within the English and mathematics sections.
[3]Note that the study did not include any content from IB "options." These are additional required components, which each IB school selects. The number of options varies by course and by subject. The IB options do not have specific standards.

2. **Depth of Knowledge Consistency:** a comparison of cognitive demand of matched content between the sets of comparison standards and the Common Core standards.

3. **Breadth of Coverage:** how broadly matched comparison standards cover content elements of the Common Core standards.

Nine English language arts experts and seven mathematics experts comprising secondary and post-secondary educators completed two tasks. Each rated the Depth of Knowledge of each standard and then determined the degree to which each comparison standard matched content in the Common Core standards. When determining match, they identified up to three Common Core standards that corresponded with a standard in a comparison set. The limit of three standards follows the Webb protocol and philosophy that alignment should concentrate only on content central to the statements.

Findings

The overall results of the study suggest substantial concurrence between the Common Core standards and the comparison standards, with somewhat greater alignment in mathematics than in ELA and literacy. For ELA and literacy, 36 of 40 analyses at the strand level meet the Categorical Concurrence criterion. For mathematics, all 25 analyses at the conceptual category level meet the Categorical Concurrence criterion.

The findings suggest general consistency between the cognitive challenge level of the Common Core standards and the five comparison standard sets. Mathematics shows somewhat more consistency of cognitive challenge than do the ELA and literacy standards. In ELA and literacy, 17 of 36 strand-level analyses indicate that the comparison standard sets are at or above the level of the Common Core standards. For mathematics, 19 of 25 conceptual category-level analyses indicate that the comparison standard sets are at or above the level of the Common Core standards. Overall, the standards from the comparison sets tend to cover the breadth of topics contained in the Common Core standards. For ELA and literacy, 37 of 40 strand analyses show strong coverage. For mathematics, findings suggest that comparison sets show strong coverage of all 25 conceptual category analyses. While every standard in the Common Core standards may not have a match with each and every set of comparison standards, the topics around which the Common Core standards are organized are reflected in the comparison standards with a high degree of frequency.

Conclusion

The business of analyzing alignment, cognitive challenge, and coverage between sets of standards remains an approximate art, even when ample care is given to controlling variation and ensuring reliable and valid results. This is one of the reasons we offer findings at a high level of aggregation. Educators should bear in mind the overall goals of the Common Core standards when considering the mechanics of alignment. Those goals are to raise the challenge level for U.S. students, to enable all students to pursue successful futures beyond high school, and to equip students with a set of core knowledge and skills that enable them to be adaptive learners throughout their lives.

Alignment is a preliminary and first step to achieving the goal of standards implementation. This study offers initial findings regarding five sets of comparison standards that are considered rigorous and good indicators of college and career readiness. While additional analyses at the individual state level are needed, the overall findings from this study suggest a general level of agreement between the Common Core standards and the comparison standards regarding what is important for high school students to know and be able to do and the cognitive level at which they need to demonstrate key skills in English language arts and mathematics in order to be ready for college and careers.

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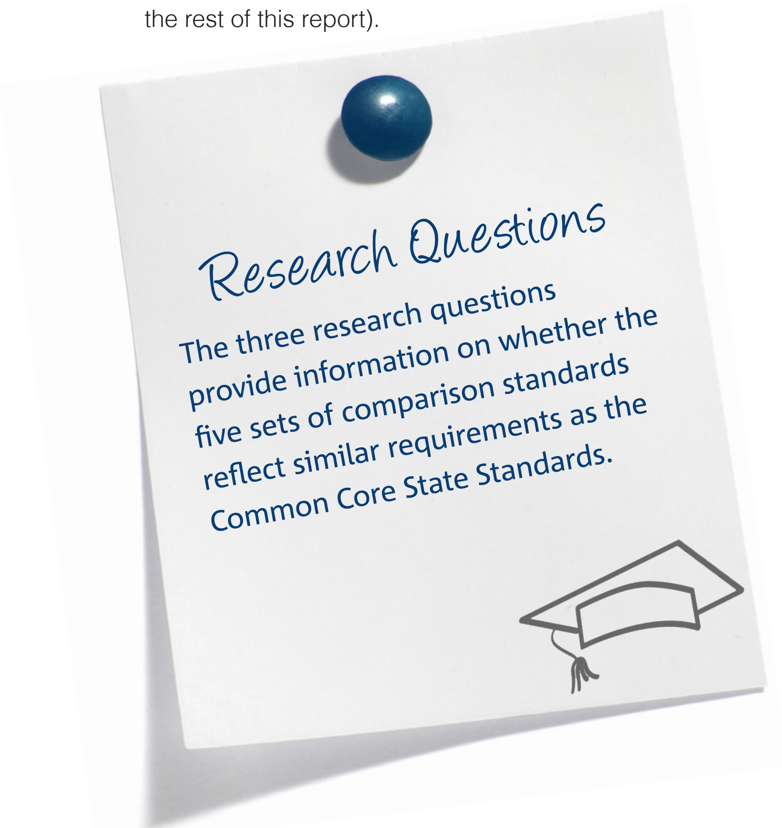
Chapter 1 | Introduction

States are concerned about the rigor of the Common Core State Standards and the relationship between this new system of standards and existing standards, particularly at the exit level from high school. The Educational Policy Improvement Center (EPIC), therefore, designed and conducted the current study to determine the extent of correspondence (alignment) between the exit level Common Core State Standards and each of five sets of existing standards. The sets of standards were selected because they were either identified as exemplary state standards, were explicitly written at the college readiness level, or represented a rigorous instructional program focused on college readiness. The purpose was to see if the Common Core State Standards covered similar content; if so, how broadly they covered the comparison standards; and whether the cognitive challenge level of aligned content was comparable.

The study addressed three research questions for each of the two subject areas for which Common Core State Standards have been developed – namely, English language arts and literacy, and mathematics. These three questions provide information on whether the five sets of comparison standards reflect similar requirements as the Common Core State Standards (called Common Core standards throughout the rest of this report).



The research questions are as follows:

1. To what extent are the knowledge and skills found in the comparison standards the same as or different from what is described in the Common Core standards? (match)
2. What is the cognitive complexity level of the Common Core standards and to what extent are the matched comparison standards at a higher or lower level of cognitive complexity? (depth)
3. How broadly do the matched comparison standards cover the content of the Common Core standards? (breadth)

The comparison standards included sets of standards from two states that have been regarded as leaders in educational standards: California and Massachusetts (Achieve, Inc., 2010). The Fordham Foundation awarded the California standards a rare A grade for both English language arts and mathematics in its 2005 and 2010 analyses (Carmichael et al., 2010). For Massachusetts, the Fordham Foundation gave the English language arts state standards A grades in its 2005 and 2010 analyses and gave the mathematics standards a B+ in 2005 and an A in 2010 (Carmichael et al., 2010). Additionally, students from Massachusetts perform among the top states on the National Assessment of Educational Performance (Lee, Grigg, & Dion, 2007; Lee, Grigg & Donahue, 2007; National Center for Education Statistics, 2009a, 2009b).

We also compared the Common Core standards to two sets of college and career readiness standards: one from a state, Texas, and another that has been used broadly during the past eight years – the Knowledge and Skills for University Success. The Texas College and Career Readiness Standards are a collection of competencies and skills that secondary students must possess to succeed in postsecondary education. They are intended to complement existing state high school standards, and create a culture of college and career readiness (Texas Education Agency, 2009). Published in 2003, the Knowledge and Skills for University Success (KSUS) standards reflect the contributions of more than 400 faculty and staff from 20 leading research universities. The faculty identified knowledge and skills that students need to succeed in entry-level courses at their institutions. Multiple peer reviews were used to hone the standards and ensure their validity (Conley, 2003). The KSUS standards have been endorsed by 28 research universities as reflecting expectations at the institutions for students in entry-level courses (Conley & Ward, 2009) and have been used by the College Board as a reference in the development of its assessments and standards (Brown & Conley, 2007).

Finally, we included a set of international standards from the International Baccalaureate (IB) Diploma Programme. Designed to be academically challenging preparation for college work, the IB program comprises six courses taken over the final two years of high school. In its analysis of the IB, the Fordham Foundation gave the IB Language A English Standard Level (SL) course a grade of B+, based on content, rigor, and clarity (Byrd, Ellington, Gross, Jago, & Stern, 2007). Reviewers deemed that the course requires students to be "well grounded in literary genres" (p. 15). They gave the Math Standard Level course a B-, praising, in particular, the free-response exams as "excellent, requiring way beyond the normal high school fare" (p. 16). The courses are known to emphasize the application of student learning so that students transfer their learning from one context to another (Phillips & Wong, 2010). Historically, the IB Diploma Programme has relied on the aims and goals specified in course documents to convey required content. Relatively recently, formal standards were developed (Conley & Ward, 2009). These standards serve as the reference point for this analysis.

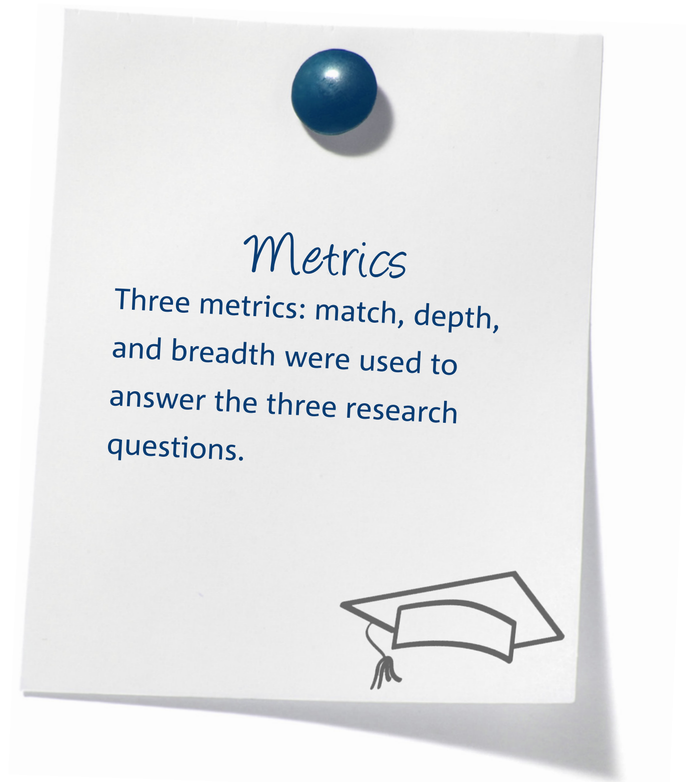
In this study, the Common Core standards are the reference point to which the other five sets of standards are compared (and we therefore refer to the other five sets of standards as the comparison standards). Our primary interest was to analyze the Common Core standards, particularly as a companion piece to another EPIC study that explored the extent to which the Common Core standards are reasonable college and career preparatory standards (see Conley, Drummond, de Gonzalez, Rooseboom, & Stout, 2011; www.epiconline.org). We used the Common Core standards as the reference point for matching to existing standards in order to identify consistent patterns of alignment or misalignment across standard sets. (Findings for the individual studies do appear throughout the report and in the appendices; however, the Common Core standards were consistently the reference point.) We did not conduct the reverse analysis: using each set of external standards as a reference for matching to the Common Core. The intent was not to provide individual alignment profiles for a number of standards sets to the Common Core. We understand many states are engaging in this type of analysis, and we did not want to duplicate such efforts.

We used Cook and Wilmes's (2007) conception of standards-to-standards alignment – a combination of linking (match between standards) and correspondence (depth and breadth) based on Webb's widely employed test-item-to-standards methodology (1997, 1999, 2002a). We used the Web Alignment Tool (WAT), created by Webb for analysis. Experts rated all the standards sets in terms of their cognitive complexity and then, following Webb's methodology (Webb, 2002b; Roach, Elliot, & Webb, 2005; Cook, 2005), matched up to three standards from Common Core standards to each standard in the comparison standard sets.

This method differs from other standards-to-standards methods in two important ways. First, the method we used limits raters to specifying three matching statements, whereas other methods allow an unlimited number of matches (e.g., Rolfus, Decker, Brite, & Gregory, 2010; Timms, Schneider, Lee, & Rolfus, 2007). Webb originally created this limitation so that reviewers would concentrate on only the central content knowledge expressed in a statement (Webb, 1999). Second, the Webb methodology (and WAT tool) assigns equal importance to every set of matches. Therefore, Webb emphasizes that when making a match, raters should ensure that the comparison statement targets, in full, the reference statement (Webb, Alt, Ely, & Vesperman, 2005). In short, the methodology does not make use of partial matches the way that other alignment studies do (e.g., Rolfhus, Cook, Brite, & Hartman, 2010). As Rolfhus, Cook, Brite, and Hartman (2010) point out, a weakness of the partial match methodology is that indication of partial matching can mean that the comparison standard matches just one element of a multidimensional reference standard or that it matches all but one of the many elements. Thus, we decided on a less ambiguous and more conservative definition of matching.

We use three metrics: match, depth, and breadth, to answer the three research questions. To answer the first research question, we look at the measure of match, Categorical Concurrence, to determine the extent to which content is consistent across the Common Core standards and each set of comparison standards. To answer the second research question, we first determine the cognitive demand of the Common Core standards and the comparison standards by means of the Depth of Knowledge rating. Then, we compare cognitive demand to gauge where cognitive challenge is higher, lower, or the same using the Depth of Knowledge Consistency statistic for content areas that match with the Common Core standards. We address the third research question using the Breadth of Coverage metric that ascertains the extent to which the comparison standards that match Common Core standards cover a similar scope of content.

In Chapter 2, we describe the methodology and provide an overview of the standards that were used in the study. Chapter 3 presents the results of alignment of the Common Core standards for English language arts and literacy, and Chapter 4 presents results for mathematics. Chapter 5 discusses study findings and implications. Appendix A provides more information about the methodology that was used. Appendix B lists the Common Core standards and summary information about the expert ratings. Appendices C and D provide more detailed information on the findings for each set of comparison standards. Lengthier and more detailed information about each set of comparison standards and precise matches with the Common Core standards is provided under separate cover in the Technical Supplement (https://www.epiconline.org/files/pdf/LiningUpTechnicalSupplement.pdf).



We measured alignment using a modified version of the Webb alignment protocol. The Webb methodology (1997; 1999; 2002a) has been widely used to assess alignment of assessments to standards. The approach relies on using panels of experts to examine content and cognitive processes in assessments and standards and then, based on those experts' ratings, calculating statistics of alignment. These statistics demonstrate how closely the assessment reflects the standards (Webb, Herman, & Webb, 2007). Over time, researchers have used modified versions of the Webb method with different variations of alignment indices (Rothman, 2003; Wixson, Fisk, Dutro, & McDaniel, 2002).

The approach has also been modified to determine standards-to-standards alignment (Cook, 2007; Cook & Wilmes, 2007). Standards-to-standards alignment provides a means to quantify and evaluate the extent of overlap between different sets of standards in terms of knowledge, skills, and content. Alignment methodology also allows for comparison between standards in terms of the cognitive demand of content and in terms of the breadth of overlap. In the current study, alignment includes three indices:

1. **Categorical Concurrence:** the extent of overlap or match between the comparison standards and the Common Core standards.
2. **Depth of Knowledge Consistency:** a comparison of cognitive demand of matched content between the sets of comparison standards and the Common Core standards.
3. **Breadth of Coverage:** how broadly matched comparison standards cover content elements of the Common Core standards.

Below, we discuss the process for collecting the requisite data for alignment and then explain the alignment statistics and data analysis.

Data Collection

We followed the training and data collection processes laid out by Webb et al. (2005).

Raters

Raters were recruited from a pool of experts who had experience with standards alignment. Representation from national educational organizations was also sought; invitations were extended to the International Reading Association and the National Council of Teachers of Mathematics.

A total of 16 raters participated, including nine English language arts (ELA) experts and seven mathematics experts. The raters comprised a mix of secondary and post-secondary educators. Each subject area had a group leader. This individual had more standards alignment experience than other raters in the subject area and helped facilitate the group.

Rater Orientation

Each group leader participated in a web conference orientation session prior to the full group sessions. EPIC researchers explained the overall alignment process and specific leader responsibilities. Then during group sessions, the leaders explained the Depth of Knowledge (DOK) ratings and coding process and guided other raters through alignment examples under the overall supervision of EPIC researchers.

Each group (ELA and mathematics) participated in two web conferences. During the first session, EPIC researchers explained the standards-to-standards alignment process and defined terminology. Under guidance from the group leader, the raters developed an initial understanding of DOK categories (explained below) and engaged in practice ratings. At the end of the first training, each group was given the task of assigning a DOK rating to each Common Core high school mathematics standard or each Common Core standard for ELA and literacy for grades 11–12.

During the second session, raters agreed upon a single DOK rating for each Common Core standard. This was accomplished by presenting all raters' DOK levels for each Common Core standard to the entire group of raters. Standards without a clear majority rating were discussed until consensus was reached. In the second part of the call, raters discussed and completed practice examples of how to determine whether to match comparison standards and Common Core standards. They developed decision rules designed to provide clarity in situations where ambiguity or confusion could arise in the ratings process. As Webb (2002b) points out, although the training process involves calibration, it is not designed for raters to reach exact agreement. Reviewers' responses are averaged, which helps account for differences in coding (Webb, 1999). Additionally, variance among reviewers reflects valid differences of opinion resulting from lack of clarity in standards statements (Webb, 2002b).

Rating Process

For each of the five sets of comparison standards, raters independently completed two tasks. They rated the DOK of each comparison standard and then determined the degree to which each comparison standard matched content in the Common Core standards. When determining match, they identified up to three Common Core standards that corresponded with a standard in a comparison set. The limit of three standards follows the Webb protocol and philosophy that alignment should concentrate only on content central to the statements. If raters found more than three statements that could match, they used professional judgment to select the three best matches. Raters were not allowed to make partial matches; that is, the comparison standard had to match the full Common Core standard. This meant that a broader comparison standard could be matched to a more narrow Common Core standard, but the reverse could not occur because the content in the Common Core standard would not be fully addressed.

If raters found that every standard within a particular Common Core subarea corresponded with comparison standards, then they rated at the superordinate level. This was the only time that more than three standards could be matched. For ease of reference, we call this level a topic for the Common Core ELA and literacy standards. The Common Core mathematics standards refer to this level as a *cluster*. Rating data were entered into the Web Alignment Tool (WAT) for analysis. Next, we present more specific information about the number of statements that were rated for each set of standards.

Rating of Common Core Standards

English Language Arts and Literacy

The Common Core English language arts (ELA) and literacy standards for grades 11–12 are organized into *strands, anchor standards*, and *standards*. We use the term *strand* to refer to each of the eight groups of related standards. Each strand is based on six to ten statements, called College and Career Readiness (CCR) Anchor Standards. The CCR Anchor Standards aim to describe cross-disciplinary literacy expectations that should be met in order for students to be prepared for success in college and workforce training programs (Common Core State Standards Initiative, 2010b). For each strand, the CCR standards present broad concepts that are applied to all grade levels. Specific standards then spell out how the CCR Anchor Standards are approached for different grade bands.[1] For this study, we used the Common Core ELA and literacy standards for the highest grade band: grades 11–12. Table 1 summarizes the organization of the Common Core ELA and literacy standards for high school.

Each of the strands contains two to four organizing categories,

[1]In some cases the high school grade-specific standard is identical to the College and Career Readiness anchor standard.

Table 1. Organization of the Common Core English Language Arts and Literacy Standards

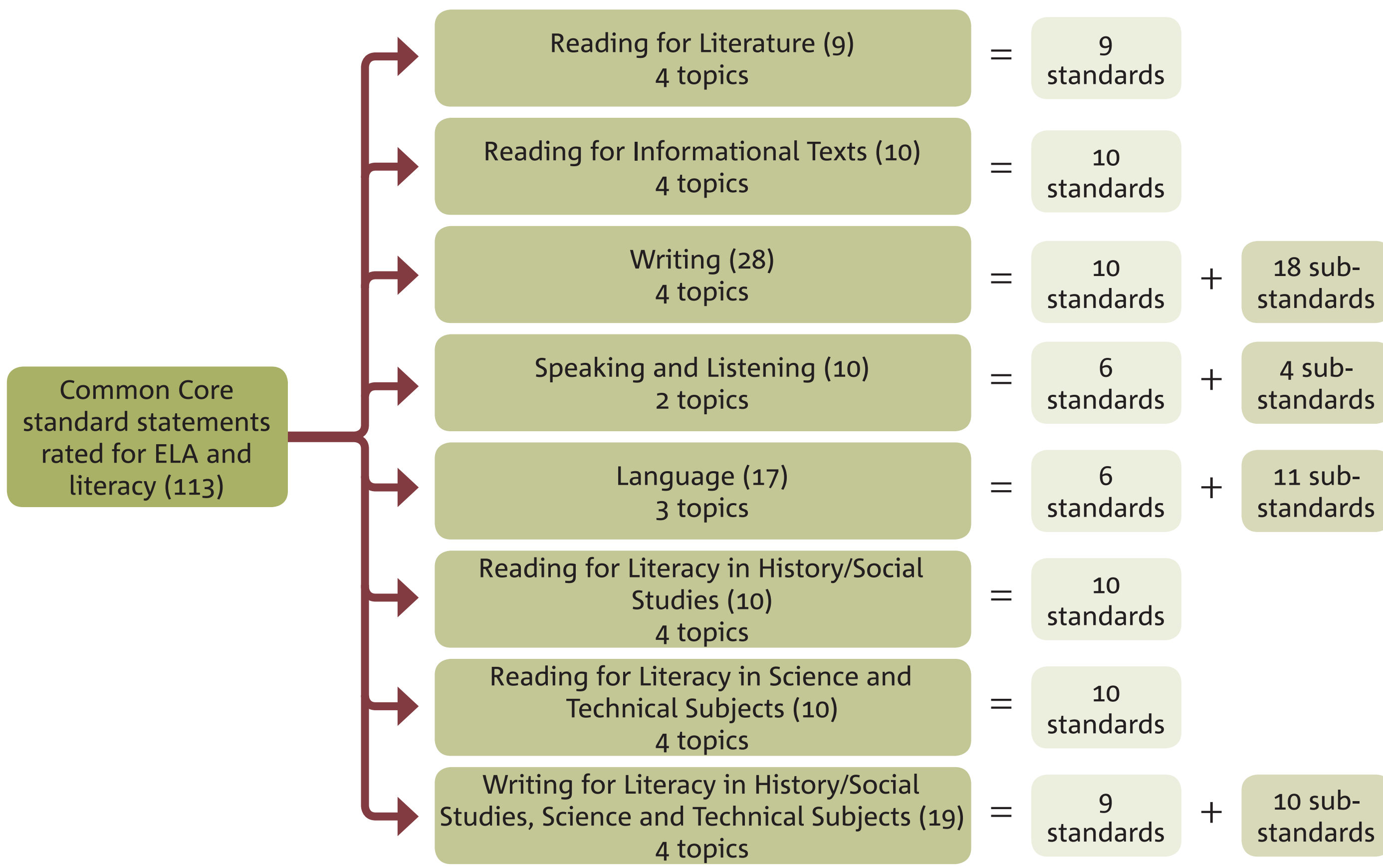
Standards for English Language Arts	Standards for Literacy
CCR Anchor Standards for Reading[a]	CCR Anchor Standards for Reading[a]
1. Reading for Literature	6. Reading in History/Social Studies
2. Reading for Informational Texts	7. Reading in Science and Technical Subjects
CCR Anchor Standards for Writing[b]	CCR Anchor Standards for Writing[b]
3. Writing	8. Writing in History/Social Studies, Science, and Technical Subjects
CCR Anchor Standards for Speaking and Listening	
4. Speaking and Listening	
CCR Anchor Standards for Language	
5. Language	

[a]The 10 reading CCR anchor standards are the same for the English language arts and literacy strands.
[b]The 10 writing CCR anchor standards are the same for English language arts and literacy strands.

which we refer to as topics. Additionally, half of the Common Core ELA strands contain statements that are organized below the standard level, as sub-standards. For the purposes of the study, these were rated as though they were on the same level as standards. Thus the Common Core standards for ELA and literacy comprised 113 ratable statements (see Figure 1).[2]

[2]For the purposes of analysis in the WAT tool, elements in the Common Core standards were organized to fit the Webb labeling structure, which uses only three levels. Common Core strands were labeled as standards, the Common Core topics were labeled as goals, and Common Core standards and sub-standards were both labeled as objectives.

Figure 1. Common Core English Language Arts and Literacy Standards for Grades 11 and 12: Number of Rated Statements



Mathematics

For the Common Core mathematics standards for high school, *conceptual categories* are the highest level of related content. There are six conceptual categories: Number and Quantity, Algebra, Functions, Modeling, Geometry, and Statistics and Probability. There is just one grade band for high school, intended for grades 9–12. The mathematics standards do not have anchor standards as do ELA and literacy. Mathematics is organized into groupings beneath the conceptual category level labeled *domains* and *clusters*. Because many mathematics topics and skills are interconnected across domains, standards from these groupings may sometimes be closely related. One of the conceptual categories – Modeling – has standards that are integrated into other conceptual categories and therefore analyses for the category were not conducted separately. (The modeling standards are indicated in the Common Core document with a star symbol (★).)

All conceptual categories contain standards with sub-standards. As with English language arts and literacy, these were rated as though they were on the same level as standards so that no information would be lost. The Common Core mathematics standards, therefore, had 192 ratable statements (see Figure 2).[3]

The Common Core high school mathematics standards aim to describe the mathematics that students should study in order to be college and career ready (Common Core State Standards Initiative, 2010c). Mathematics standards marked with a (+) represent advanced content and are intended to prepare students for higher-level courses. While not all students would be expected to complete these standards, their content might appear in courses designed for all students. The advanced content standards appear throughout the domains and were rated as all other standards.

[3]For the purposes of the study, Common Core elements were organized to fit the Webb labeling structure, which uses three levels. Common Core conceptual categories were labeled as standards, Common Core clusters were labeled as goals, and Common Core standards and sub-standards were labeled as objectives. The Common Core domain label was not used in the Webb structure, as it is a broad description of the more detailed levels that follow.

Figure 2. Common Core Mathematics Standards for High School: Number of Rated Statements

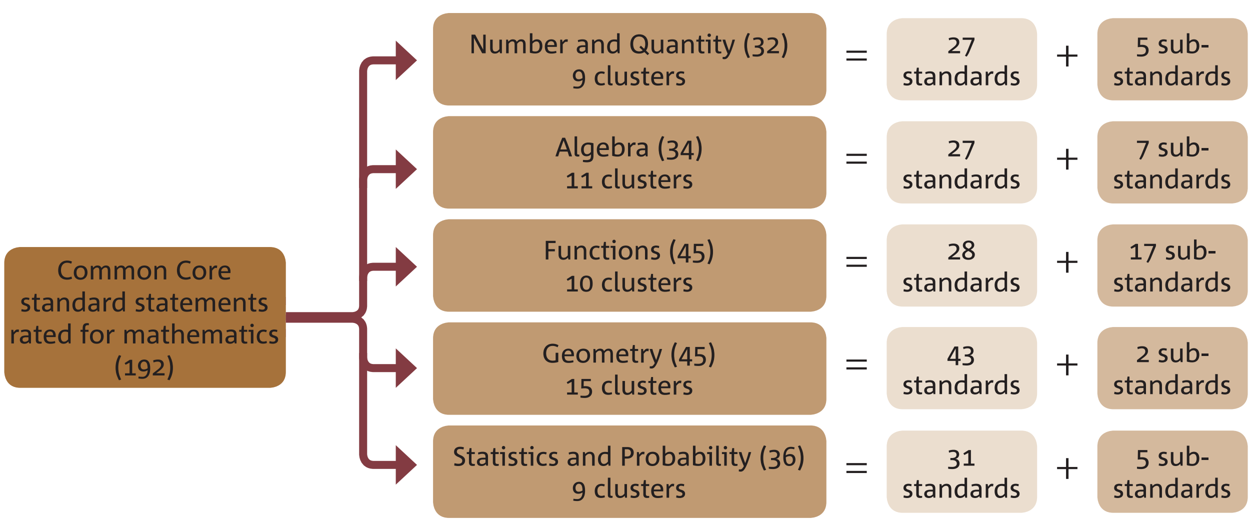


Table 2. Descriptive Information About the Five Comparison Standards Sets for English Language Arts (ELA) and Literacy

ELA comparison standards sets	Number of comparison standards
California ELA standards for grades 11–12	108
Massachusetts ELA standards for grades 11–12	41
Texas ELA and cross-disciplinary college and career readiness standards	89 (45 are cross-disciplinary)
Knowledge and Skills for University Success ELA college and career readiness standards	73
International Baccalaureate ELA standards for grades 10–12	49

The Common Core mathematics standards include eight Standards for Mathematical Practice. These hone in on "processes and proficiencies" with longstanding importance in mathematics education (Common Core State Standards Initiative, 2010c), such as problem solving, reasoning and proof, communication, representation, adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and the inclination to see mathematics as sensible and useful. The Standards for Mathematical Practice are explained at the beginning of the Common Core standards document and then presented again on each conceptual category overview page. In general, because they are not laid out as specific standards, the practices were not rated in this study. However, one of the practices is making mathematical models and specific modeling standards appear in the Common Core document.

Table 3. Descriptive Information About the Five Comparison Standards Sets for Mathematics

Mathematics comparison standards sets	Number of comparison standards
California mathematics standards for grades 8–12	185
Massachusetts mathematics standards for grades 9–12	103
Texas mathematics and cross-disciplinary college and career readiness standards	115 (45 are cross-disciplinary)
Knowledge and Skills for University Success mathematics college and career readiness standards	83
International Baccalaureate mathematics standards for grades 10–12	189

Rating of Comparison Standards

The five sets of comparison standards are as follows:

- The Content Standards for California Public Schools, for the 11^{th}–12^{th}-grade band in English language arts and for the 8^{th}–12^{th}-grade band in mathematics (1997)
- The Massachusetts Curriculum Frameworks, for the 11^{th}–12^{th}-grade band in English language arts (2001) and mathematics (2000)
- The Texas College and Career Readiness Standards in English language arts, mathematics, and cross-disciplinary standards (2008)
- The Knowledge and Skills for University Success (KSUS) standards in English and mathematics,[4] developed as college-preparatory standards by Standards for Success
- The International Baccalaureate (IB) Diploma Programme English language arts and mathematics standards, developed by EPIC, for IB's Programs of Study for 10^{th}–12^{th}-grades[5]

Among the five sets, the number of comparison standards in ELA and literacy ranged between 41 and 108. The number of comparison mathematics standards ranged between 83 and 189. In some instances, the lowest level for rating was not the very smallest grain size within the sets of comparison standards but rather the level that had consistent detail for the purpose of alignment. Tables 2 and 3 break down the number of ratable statements for each of the comparison standards sets for ELA and literacy and mathematics.

[5]These did not include the Knowledge and Skills Foundations within the English and mathematics sections.
[6]Note that the study did not include any content from IB "options." These are additional required components, which each IB school selects. The number of options varies by course and by subject. The IB options do not have specific standards.

Rater Reliability

Reliability statistics were in the acceptable range for all Depth of Knowledge ratings. For each set of comparisons we examined the intraclass correlation, which calculates agreement, accounting for the possibility of agreement by chance and ratings that are close but not an exact match (Shrout and Fleiss, 1979). In this context, values of .60 or larger indicate reasonable level of agreement, and values of .70 or larger indicate a good level of reliability (Cook & Wilmes, 2007). For ELA and literacy, the intraclass correlation was close to .70 for one comparison and above .85 for the other four comparisons. The correlations range from .68 to .94 (see Table 4).

For mathematics, the intraclass correlations were close to or above .80 for each set, ranging from .78 to .87 (see Table 5).

Rater consistency data were examined for variance and errant coding to determine if any single rater was skewing reliability and therefore warranted removal from the analysis data. No such pattern was found and all data from all raters were retained.

Table 4. Consistency of Depth of Knowledge Ratings Across Nine Raters for English Language Arts and Literacy

Comparison standard set	Number of standards	Intraclass correlation
California	108	0.91
Massachusetts	41	0.93
Texas and College and Career Readiness	89	0.86
Knowledge and Skills for University Success	73	0.94
International Baccalaureate	49	0.68

Table 5. Consistency of Depth of Knowledge Ratings Across Seven Raters for Mathematics

Comparison standard set	Number of standards	Intraclass correlation
California	185	0.87
Massachusetts	103	0.83
Texas and College and Career Readiness	115	0.81
Knowledge and Skills for University Success	83	0.85
International Baccalaureate	189	0.78

Figure 3. Illustration of Categorical Concurrence Statistic

Data Analysis

Categorical Concurrence

Webb's Categorical Concurrence statistic measures the average number of comparison standards from one set of standards that raters match to specific standards from the reference set of standards. In the current study, Categorical Concurrence is the mean number of matches between each set of comparison standards and the Common Core standards. It is measured for each content-specific area of the Common Core standards, namely the eight strands in English language arts and literacy and the five conceptual categories in mathematics. If, on average across all raters, at least one standard in the content-specific area of the Common Core was matched to a standard in the comparison standard set, the criterion for that content-specific area is met. In test items-to-standards alignment, at least six test items must match in order to meet the criterion (Webb et al., 2005). This number is necessary due to the specificity of test items, in that no single test item is expected to comprehensively assess an entire standard. In standards-to-standards alignment, however, standards tend to be broad statements describing skills and knowledge, and a single correspondence between two sets of standards for a content area is considered sufficient to meet the criterion (Cook, 2005). Figure 3 illustrates this criterion graphically.

Depth of Knowledge Consistency

Depth of Knowledge (DOK) is determined by having all raters assign one of four levels of cognitive complexity (Webb, 1997) to each Common Core standard and each standard in the five comparison standards sets. Webb outlined the following levels:

Level 1: Recall and reproduction

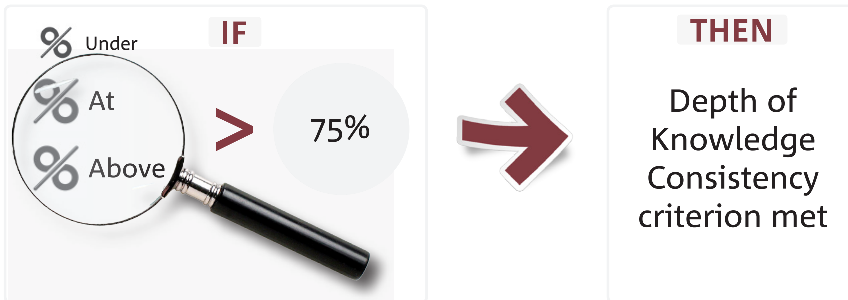
Level 2: Skills and concepts

Level 3: Strategic thinking

Level 4: Extended thinking

Level 1 content and skills require recall of information, such as reciting a fact or performing a simple procedure. Level 2 skills involve more than one mental step, such as making a decision on how to approach a problem before solving it. In Level 3, the cognitive demands are complex and abstract; the content requires deep understanding exhibited through planning and use of evidence. Level 4 content, with the highest cognitive demand, requires students to make connections, relate ideas, and select one approach among many alternatives (Webb et al., 2005).

Figure 4. Illustration of Depth of Knowledge Consistency Statistic

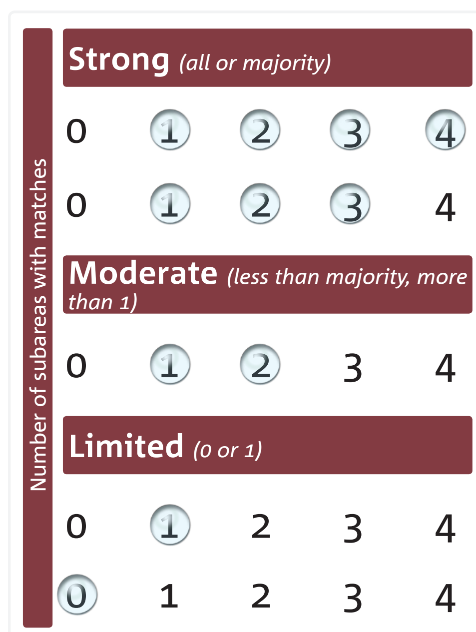


Once raters gave all standards a DOK rating, we calculated the DOK Consistency statistic for each Common Core content-specific area. The statistic involves comparing the DOK ratings of Common Core standards that had been matched to comparison standards and the DOK ratings of the standards that had been matched. We set an a priori threshold of 75% for the DOK Consistency criterion. This means that, for each Common Core content-specific area, if more than 75% of the matched comparison standards were at or above the DOK of the corresponding Common Core standard, then the criterion is met. For test items-to-standards alignment, researchers typically use a 50% DOK criterion (Webb, 1999) based on the fact that a substantial number of test items are supposed to reflect the standards, and greater match is required for categorical concurrence (Cook, 2005). For standards-to-standards comparisons, since the match requirement is less, the 50% criterion is insufficient. Therefore, a 75% DOK criterion has typically been adopted for standards-to-standards alignment (Cook, 2005). Figure 4 shows this graphically. For information on the specific details of this calculation, see Appendix A.

Breadth of Coverage

Breadth of Coverage is a measure of dispersion that shows the extent to which matched comparison standards cover a similar scope of content as that found in the reference standards. Specifically in this study, the statistic was used to calculate the number of content elements (subareas) in the Common Core standards that are covered by matched standards in the comparison standard set. For ELA and literacy, we refer to the subareas as topics. For mathematics, the subareas are referred to as clusters in the Common Core standards. When more subareas[6] have matching standards, then greater breadth of coverage exists. Specifically, the coverage statistic uses the number of subareas within a Common Core content-specific area that have standards matched with comparison standards. A strong coverage score for a content-specific area means a majority of the subareas in that Common Core content-specific area match with the comparison standard set. Moderate coverage means that more than one subarea matches for the content-specific area. Limited coverage means that within a content-specific area, one or no subarea matches to the Common Core standards. Figure 5 illustrates this criterion.

Figure 5. Illustration of Coverage Statistic



[6]Webb uses the term "goal" here as a generic term. To reduce confusion, we use terms specifically related to the organization of the Common Core standards.

Chapter 3 | Results for English Language Arts and Literacy

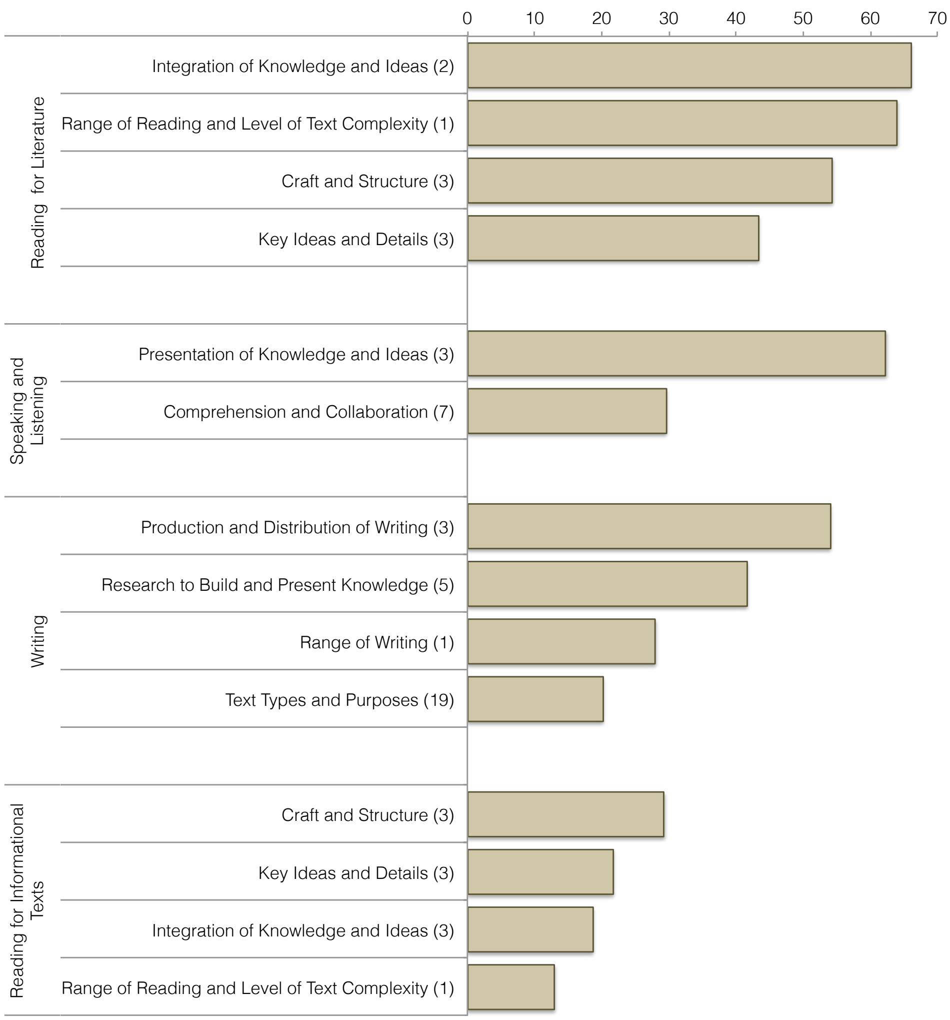
Results below describe the match between the Common Core English language arts (ELA) and literacy standards and the five sets of comparison standards. Because the Webb methodology calculates alignment at the highest level, results are generally presented at the strand level.

Match

Overall Match of Comparison Standards to the Common Core Standards

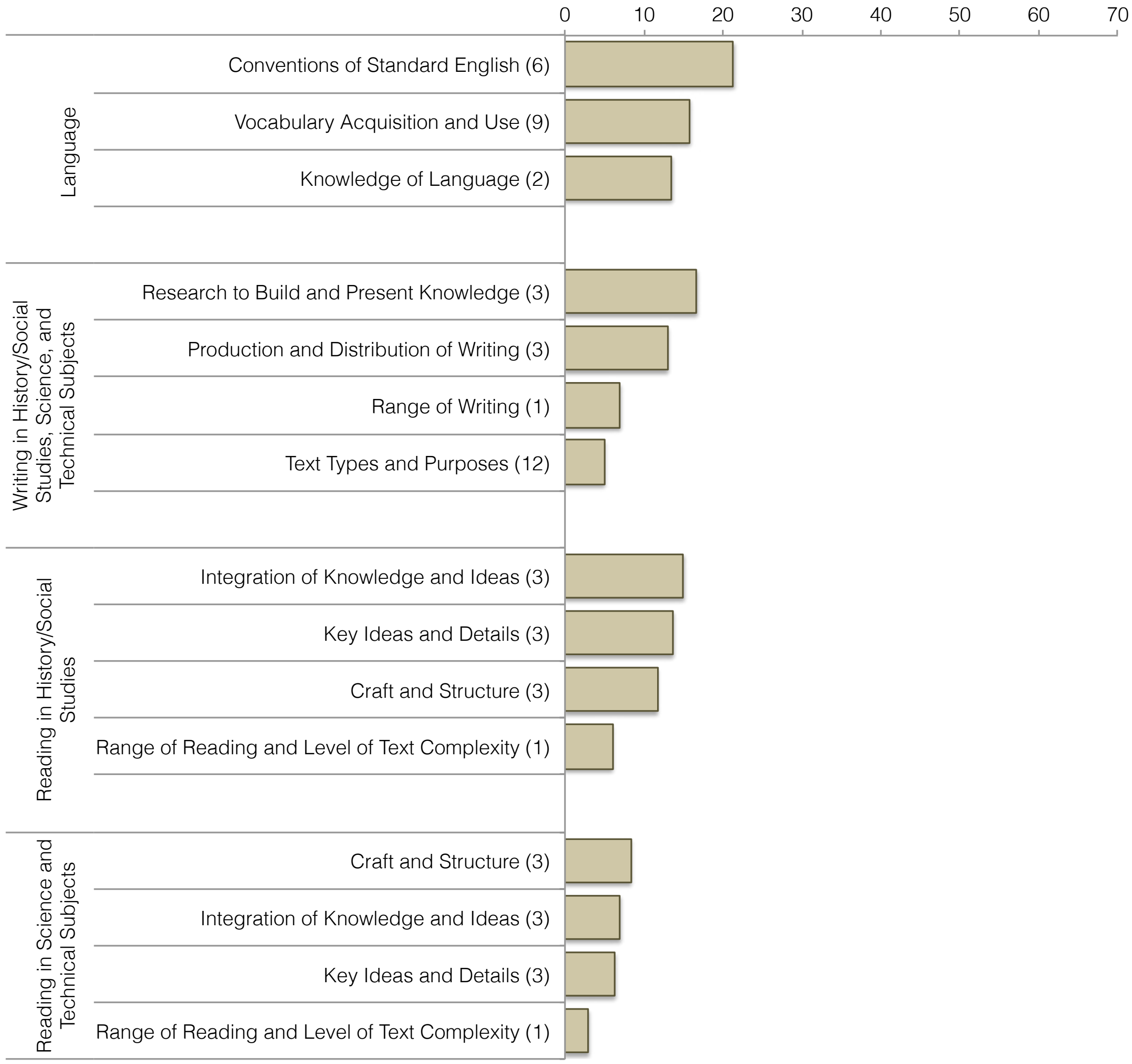
Figures 6 and 7 show the average number of matches per Common Core standard in each topic across all comparison standards. Both the strands and topics within a strand are sorted in descending order according to frequency. Figure 6 shows the first four strands and Figure 7 shows the second four strands. The graphs reveal certain tendencies between strands (e.g., more comparison standards matched for Reading for Literature than for Reading for Informational Texts). They also show trends within strands (e.g., one of the topics in Speaking and Listening has many more matching standards than the other topic).

Figure 6. Average Number of Matches per Standard Across All Comparison Standards, by Topic



Note: Numbers in parentheses indicate how many standards statements appear within the topic.

Figure 7. Average Number of Matches per Standard Across All Comparison Standards, by Topic



Note: Numbers in parentheses indicate how many standards statements appear within the topic.

Match for Each Comparison Set to the Common Core Standards

Table 6 presents the average number of matches per strand between the Common Core standards for each set of comparison standards. The data illustrate that there is a wide range in the number of average matches and that some strands have high standard deviations, indicating that raters varied widely in the number of matches they identified.

Table 6. Average Number of Matches for English Language Arts (ELA) and Literacy Common Core Strands Across All Comparison Standards

Common Core ELA and literacy strands	Average number of matches									
	CA		MA		TCCRS		KSUS		IB	
	M	SD	M	SD	M	SD	M	SD	M	SD
Reading for Literature	15.7	10.93	10.0	4.00	10.0	5.85	10.8	8.24	8.0	4.00
Reading for Informational Texts	6.0	3.74	5.0	2.32	9.9	3.21	2.7	4.09	1.1	1.41
Writing	20.4	11.88	8.6	2.83	21.8	10.23	18.9	9.00	17.1	8.35
Speaking and Listening	16.9	11.03	5.0	2.83	14.7	6.78	3.0	4.86	4.2	3.96
Language	4.9	1.97	8.2	2.39	8.6	4.27	11.1	7.36	0.2	0.78
Reading in History/Social Studies	6.2	5.24	0.6	1.21	3.6	3.23	2.3	2.69	1.4	3.38
Reading in Science and Technical Subjects	1.0	1.79	0.2	0.78	4.7	2.91	1.0	3.46	0.7	1.94
Writing in History/Social Studies, Science, and Technical Subjects	2.2	3.03	1.0	2.45	6.1	6.44	5.9	17.15	2.1	6.98
Total	**73.3**	**44.14**	**73.3**	**44.14**	**79.2**	**28.02**	**55.7**	**37.15**	**34.9**	**19.51**

CA = California; MA = Massachusetts; TCCRS = Texas College and Career Readiness Standards; KSUS = Knowledge and Skills for University Success; IB = International Baccalaureate.

Categorical Concurrence Statistic

Recall that a single correspondence in a content area between two sets of standards is considered sufficient to meet the standards-to-standards alignment criterion (Cook, 2005). Thus, overall the criterion for Categorical Concurrence between the Common Core and the five sets of comparison standards is met frequently. On average across raters, at least one standard from every set of the five comparison standards is matched to a standard in all Common Core strands, with few exceptions. This indicates overlap between the knowledge and skills specified in the Common Core standards and the content of the comparison standards. Table 7 shows the Common Core strands for which the Categorical Concurrence criterion is met across the five sets of comparison standards.

Table 7. Categorical Concurrence for English Language Arts (ELA) and Literacy

Common Core ELA and literacy strands	CA	MA	TCCRS	KSUS	IB
Reading for Literature	●	●	●	●	●
Reading for Informational Texts	●	●	●	●	●
Writing	●	●	●	●	●
Speaking and Listening	●	●	●	●	●
Language	●	●	●	●	○
Reading in History/Social Studies	●	○	●	●	●
Reading in Science and Technical Subjects	●	○	●	●	○
Writing in History/Social Studies, Science, and Technical Subjects	●	●	●	●	●

CA = California; MA = Massachusetts; TCCRS = Texas College and Career Readiness Standards; KSUS = Knowledge and Skills for University Success; IB = International Baccalaureate.

● Meets the criterion for Categorical Concurrence (averaged across raters, a single correspondence exists between the two sets of standards);
○ Does not meet the criterion for Categorical Concurrence.

The criterion is met for all (eight of eight) Common Core ELA and literacy strands for the California standards, Texas College and Career Readiness Standards, and the Knowledge and Skills for University Success. The criterion is met for six of eight Common Core ELA and literacy strands for the Massachusetts and International Baccalaureate standards.

For the Massachusetts standards, the criterion is not met for the Common Core Reading Standards in History/Social Studies and the Reading Standards in Science and Technical Subjects. Table 6 shows why the criterion is not met for each of these two areas. For the Massachusetts standards, there is an average of 0.6 standards (*SD* = 1.21) that match with the Reading Standards in History/Social Studies. There is an average of 0.2 standards (*SD* = .78) that match with the Reading Standards in Science and Technical Subjects.

For the International Baccalaureate standards, the criterion is not met for the Common Core Language Standards and Reading Standards in Science and Technical Subjects. There is an average of 0.2 standards (*SD* = .78) that match with the Common Core Language Standards and an average of 0.7 standards (*SD* = 1.94) that match with Reading Standards in Science and Technical Subjects.

Among the sets of comparison standards for which all criteria are met, there is a wide range in the number of average matches (see Table 6). As an example, for the California standards, raters identified an average of 1.0 standard (*SD* = 1.79) that matches with the Common Core Reading Standards in Science and Technical Subjects and an average of 20.4 standards (*SD* = 11.88) that match with the Writing Standards. Both of these meet the criterion.

In Appendix C, there is more detailed discussion about each set of comparison standards and how it aligns to the Common Core. The precise correspondence of matches between the Common Core standards and individual comparison sets of standards is provided in the Technical Supplement.

Depth of Knowledge

Ratings for Common Core Standards

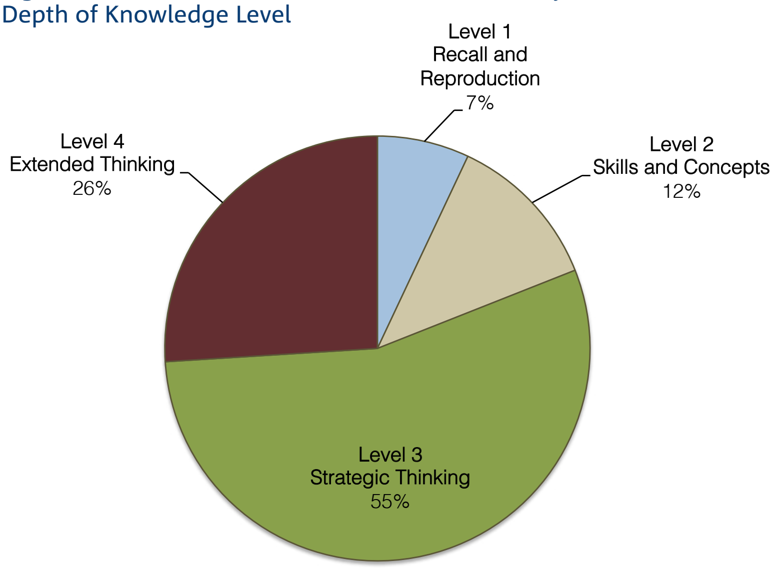
Table 8 shows the modal Depth of Knowledge (DOK) rating for each of the Common Core English language arts (ELA) and literacy strands. Most represent the strategic thinking level of cognitive demand (level 3). As the table shows, the Language Standards have lower ratings than the other strands (mode of level 2). This might be because the strand contains elements of grammar, conventions, and usage, which do not tend to involve abstract and complex thinking.

Table 8. Depth of Knowledge (DOK) Modal Ratings for English Language Arts and Literacy Common Core Strands

Common Core ELA and literacy strand	DOK mode
Reading for Literature	3
Reading for Informational Texts	3
Writing	3
Speaking and Listening	3
Language	2
Reading in History/Social Studies	3
Reading in Science and Technical Subjects	3
Writing in History/Social Studies, Science, and Technical Subjects	3

At the standard level, the Common Core standards fall across the full range of the DOK scale, but less frequently at the lower levels of the scale. Overall, only 7% of the standards are at a level 1 (recall and reproduction), 12% are at a level 2 (skills and concepts), 55% are at a level 3 (strategic thinking), and 26% are at level 4 (extended thinking). This summary information is presented graphically in Figure 8. DOK ratings for all standards are provided in Appendix B.

Figure 8. Percent of Common Core ELA and Literacy Standards at each Depth of Knowledge Level



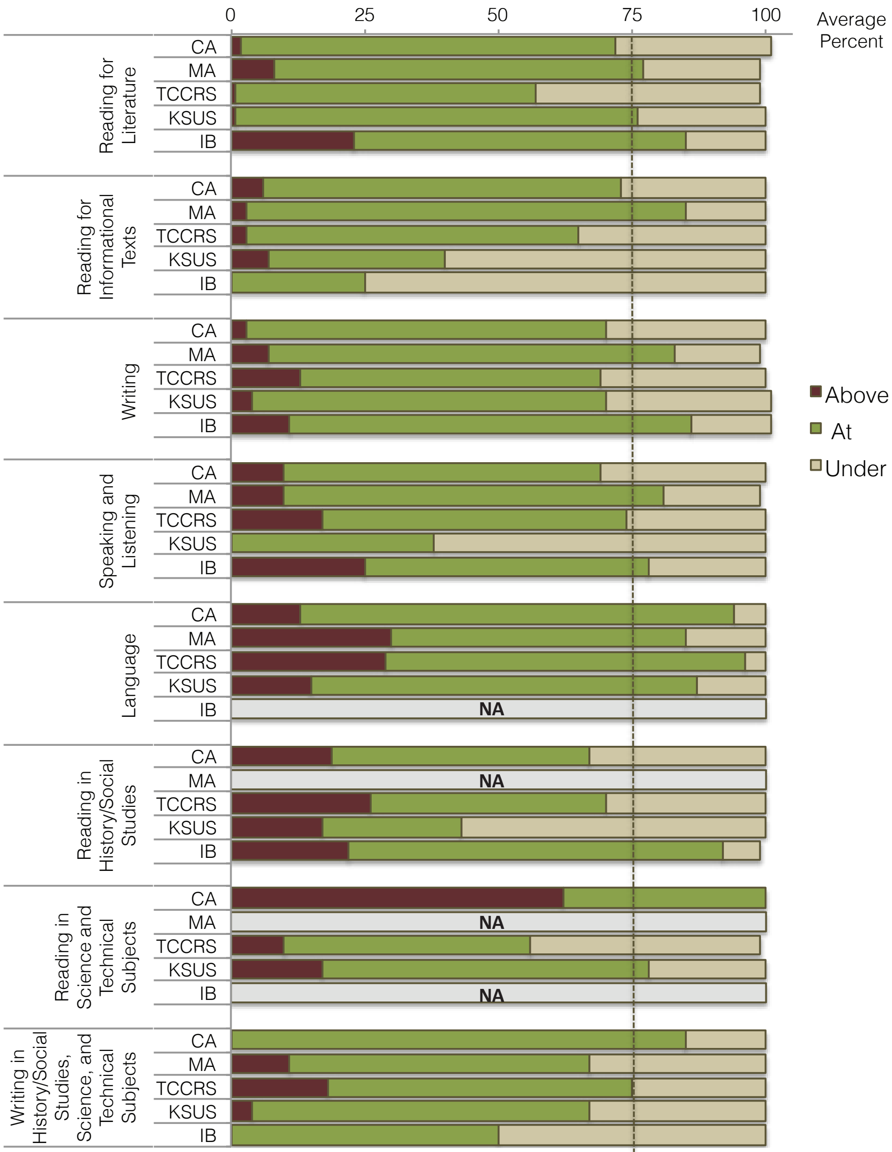
DOK Consistency of Each Comparison Set to the Common Core Standards

DOK Consistency describes the extent to which the ELA and literacy standards that experts matched between the comparison sets and the Common Core are consistent in level of cognitive demand. Meeting this criterion requires more than 75% of the comparison standards matching for the content area to be at or above the level of cognitive demand of the Common Core standards. Consistency is only computed for those content-specific areas for which the Categorical Concurrence criterion is met.

Figure 9 shows, for each comparison set, the percent of standards (averaged across raters; see Appendix A) that fell above, at, and under the cognitive demand level of the Common Core standards. In the figure, a line is drawn at 75%, which is the criterion for the percent of standards falling above or at the cognitive demand level of the Common Core in order to meet the DOK Consistency statistic. Findings for the DOK Consistency statistic are inconsistent across the different comparison sets and different strands. Overall, the Common Core strand for which the DOK criterion is most often met is the Language Standards strand, indicating that the matched comparison standards are at or above the DOK of the Common Core Language Standards. The strands for which the criterion is met for less than half of the comparison standards sets are the Reading Standards for Informational Texts, the Writing Standards, the Speaking and Listening Standards, the Reading Standards in History/Social Studies, the Reading Standards in Science and Technical Subjects, and the Writing Standards in History/Social Studies, Science, and Technical Subjects. For these strands, there are more comparisons for which the Common Core standards are rated at a higher level in terms of cognitive demand. As Figure 9 shows, some standards sets fall just above or just below 75% for some strands and results would be different if an alternative criterion had been selected.

Looking at DOK Consistency across the comparison sets of standards, the criterion is met most often for the Massachusetts standards (for five out of six strands that fulfill the Categorical Concurrence criterion). This suggests that for ELA and literacy, the cognitive demand of the Massachusetts standards is similar to or more cognitively complex than the Common Core standards. The International Baccalaureate standards meet the criterion for four out of six strands that fulfill the Categorical Concurrence criterion. For the other three sets (California, Texas College and Career Readiness, and Knowledge and Skills for University Success), the criterion is met for two or three of eight strands that fulfill the Categorical Concurrence criterion.

Figure 9. Depth of Knowledge Level Comparisons of the Five Sets of Comparison Standards to Common Core ELA and Literacy Standards



Note: Total percent may equal slightly above or below 100 due to rounding.

Breadth of Coverage

Breadth of Coverage gives information on the breadth or dispersion of corresponding standards across standards sets. A comparison set of standards could have a high degree of concurrence with reference standards but still not cover the entire scope or breadth of content of the reference standards. In this study, the criterion is related to the number of Common Core topics within a strand for which a set of comparison standards has matches to the Common Core standards. Strong coverage means that a majority of topics in the strand have matches. Moderate coverage means that more than one topic in the strand has matches. Limited coverage means that one or fewer than one topic in the strand has matches. If coverage is limited, this indicates that comparison content tends to concentrate in particular topics of the Common Core strand, according to the way the Common Core standards are organized. Therefore, other categories of content are not covered by the comparison standards.

For the most part, coverage between the Common Core standards and the comparison standards was consistently strong or broad (see Table 9). Most sets of comparison standards cover a majority of Common Core topics for each strand. There were a few exceptions. For the Massachusetts standards, the Reading Standards in Science and Technical Subjects strand has moderate coverage. For the International Baccalaureate standards, the Reading Standards for Informational Texts strand also has moderate coverage and the Language Standards strand has limited coverage. Two of these (the Massachusetts reading strand and the International Baccalaureate language strand) also do not meet the Categorical Concurrence criterion, which suggests that the lower number of average matches for these strands may have limited coverage. However, this is not a necessary relationship because there are also two strands for which the Categorical Concurrence criterion is not met but that still have strong coverage (Reading Standards in History/Social Studies for Massachusetts and Reading Standards in Science and Technical Subjects for International Baccalaureate).

Appendix D presents more information about the distribution of standard matches within each of the eight ELA and literacy strands.

Table 9. Breadth of Coverage Criterion for English Language Arts and Literacy

Common Core ELA and literacy strand	CA	MA	TCCRS	KSUS	IB
Reading for Literature	🟢	🟢	🟢	🟢	🟢
Reading for Informational Texts	🟢	🟢	🟢	🟢	⚪
Writing	🟢	🟢	🟢	🟢	🟢
Speaking and Listening	🟢	🟢	🟢	🟢	🟢
Language Standards	🟢	🟢	🟢	🟢	🔴
Reading in History/Social Studies	🟢	🟢	🟢	🟢	🟢
Reading in Science and Technical Subjects	🟢	⚪	🟢	🟢	🟢
Writing in History/Social Studies, Science, and Technical Subjects	🟢	🟢	🟢	🟢	🟢

CA = California; MA = Massachusetts; TCCRS = Texas College and Career Readiness Standards; KSUS = Knowledge and Skills for University Success; IB = International Baccalaureate.
🟢 means that a majority of topics in the Common Core strand have at least one matched comparison standard;
⚪ means that one topic in the Common Core strand have at least one matched comparison standard;
🔴 means that one or fewer than one topic in the Common Core strand have at least one matched comparison standard.

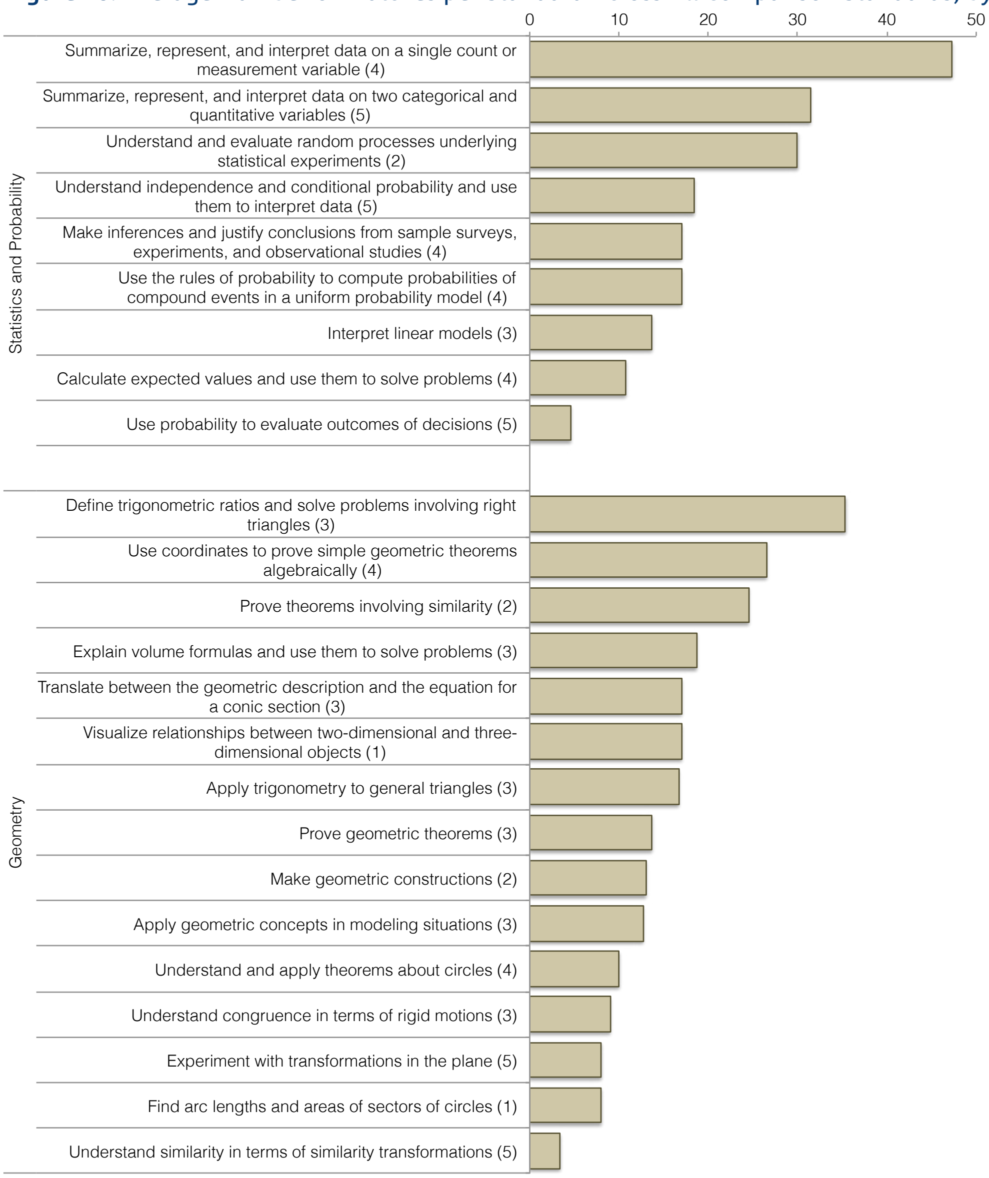
Results below describe the match between the Common Core mathematics standards and the five sets of comparison standards. Results are generally presented at the conceptual category level.

Match

Overall Match of Comparison Standards to the Common Core Standards

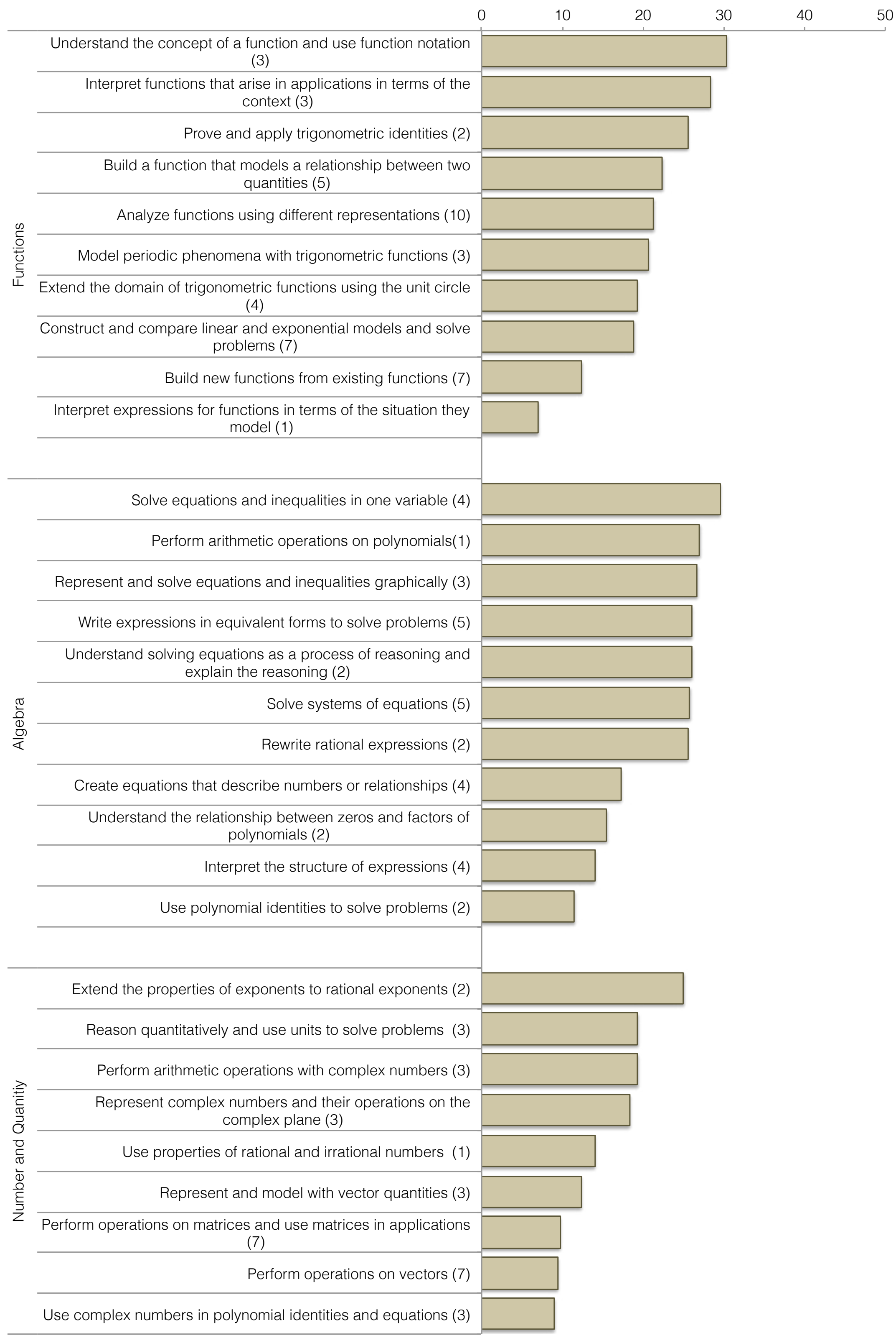
Figures 10 and 11 show the average number of matches per Common Core standard in each cluster across all comparison standards. Both the conceptual categories and clusters within a category are sorted in descending order according to frequency. Figure 10 provides the information for Statistics and Probability, and Geometry. Figure 11 provides the information for Functions, Algebra, and Number and Quantity. The graphs show tendencies between and within conceptual categories.

Figure 10. Average Number of Matches per Standard Across All Comparison Standards, by Cluster



Note: Numbers in parentheses indicate how many standards statements appear within the cluster.

Figure 11. Average Number of Matches per Standard Across All Comparison Standards, by Cluster



Note: Numbers in parentheses indicate how many standards statements appear within the cluster.

Match for Each Comparison Set to the Common Core Standards

Table 10 shows the average number of matches per conceptual category between the Common Core standards and each set of comparison standards for mathematics. Generally, all categories for all standards sets have a relatively high number of matches. As with English language arts and literacy, some categories have very high standard deviations, indicating that raters varied widely in the number of matches they identified.

Categorical Concurrence Statistic

Because a single correspondence in a content area between two sets of standards is considered sufficient to meet the standards-to-standards alignment criterion (Cook, 2005), the criterion for Categorical Concurrence is met for every conceptual category for every comparison standard set. Across raters, more than one standard from every set of the five comparison standards on average is matched to a Common Core standard (see Table 11).

In Appendix C, there is more detailed discussion for each set of comparison standards and how it aligns to the Common Core. The precise correspondence of matches between the Common Core standards and individual comparison sets of standards is provided in the Technical Supplement.

Table 10. Average Number of Matches for Mathematics Common Core Standards Across Five Sets of Standards

Common Core mathematics conceptual category	Average number of matches									
	CA		MA		TCCRS		KSUS		IB	
	M	SD	M	SD	M	SD	M	SD	M	SD
Number and Quantity	15.9	9.85	16.7	9.07	6.9	4.05	5.0	4.95	17.4	9.75
Algebra	32.4	22.23	23.1	10.47	17.9	19.54	19.0	18.35	17.0	12.66
Functions	25.9	18.44	27.7	14.70	21.1	11.75	17.4	13.32	38.9	23.27
Geometry	26.3	18.76	24.0	14.81	18.9	10.16	14.9	11.01	12.0	7.73
Statistics and Probability	21.1	14.00	17.4	8.78	31.9	19.69	4.6	2.61	30.9	18.47
Total	**121.6**	**81.18**	**109.0**	**56.23**	**96.6**	**60.86**	**60.9**	**45.49**	**116.1**	**65.73**

CA = California; MA = Massachusetts; TCCRS = Texas College and Career Readiness Standards; KSUS = Knowledge and Skills for University Success; IB = International Baccalaureate.

Table 11. Categorical Concurrence for Mathematics

Common Core mathematics conceptual category	CA	MA	TCCRS	KSUS	IB
Number and Quantity	●	●	●	●	●
Algebra	●	●	●	●	●
Functions	●	●	●	●	●
Geometry	●	●	●	●	●
Statistics and Probability	●	●	●	●	●

CA = California; MA = Massachusetts; TCCRS = Texas College and Career Readiness Standards; KSUS = Knowledge and Skills for University Success; IB = International Baccalaureate.
● Meets the criterion for Categorical Concurrence (averaged across raters, a single correspondence exists between the two sets of standards);
● Does not meet the criterion for Categorical Concurrence.

Depth of Knowledge

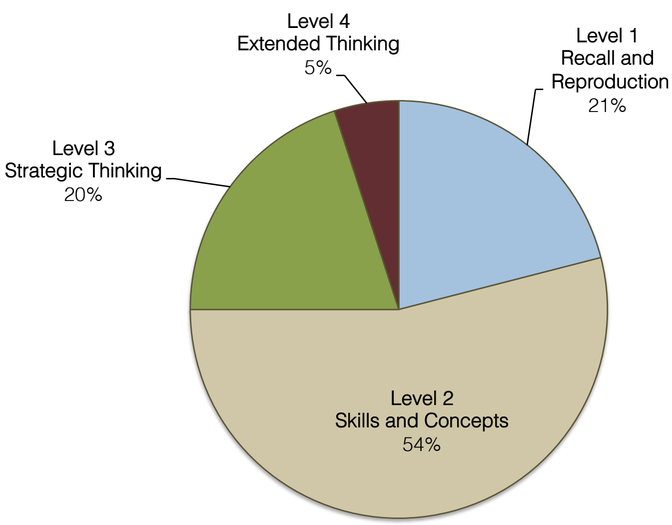
Ratings for Common Core Standards

Table 12 shows the modal Depth of Knowledge (DOK) rating for each of the Common Core mathematics conceptual categories. Three categories represent the skills and concepts level of cognitive demand (level 2). In the Number and Quantity category, which has a mode of level 1 (recall and reproduction), the standards involve such skills as explaining properties (of irrational exponents, rational and irrational numbers) and performing arithmetic operations (with complex numbers, vectors, and matrices). These skills may be more oriented toward one-step recall or problem solving. In the Geometry category, the standards involve such skills as proving theorems, deriving equations, and using experimentation to verify concepts such as similarity. Such skills may lend themselves more toward strategic and extended thinking, which could explain why the mode rating for that content-specific area was at level 3 (strategic thinking).

Table 12. Depth of Knowledge (DOK) Modal Ratings for Mathematics Common Core Conceptual Categories

Common Core mathematics conceptual category	DOK mode
Number and Quantity	1
Algebra	2
Functions	2
Geometry	3
Statistics and Probability	2

Figure 12. Percent of Common Core Mathematics Standards at each Depth of Knowledge Level



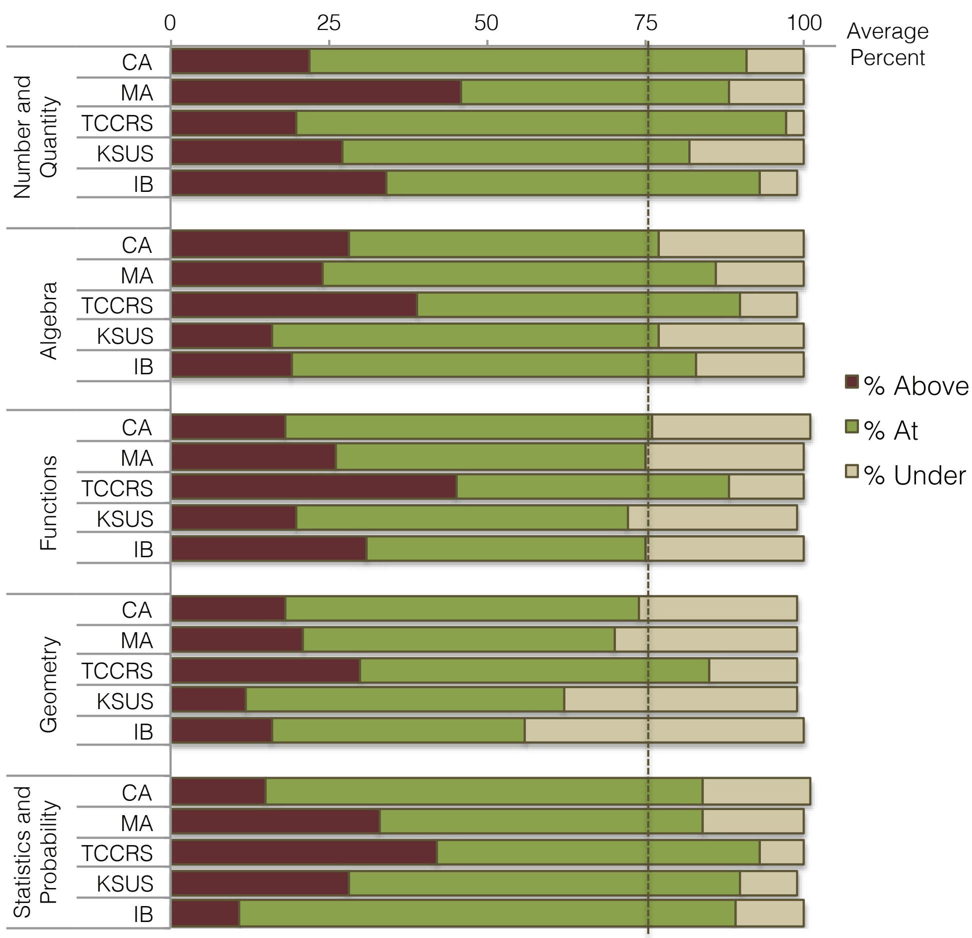
At the standard level, the Common Core mathematics standards cover a somewhat broader range of cognitive complexity than the ELA and literacy Common Core standards with fewer at the highest level. The majority of the Common Core mathematics standards are rated as skills and concepts (level 2). Overall, 21% are at a level 1 (recall and reproduction), 54% are at a level 2 (skills and concepts), 20% are at a level 3 (strategic thinking), and 5% are at level 4 (extended thinking). Figure 12 summarizes this information. DOK ratings for all standards are provided in Appendix B.

As mentioned earlier, the Standards for Mathematical Practice within the Common Core standards were not rated for the purposes of this study. The Mathematical Practices are designed for educators to apply across all Common Core mathematics standards. Thus, using the mathematics practices in the application and use of the other Common Core mathematics standards could conceivably lead to different levels of cognitive demand than those presented here.

DOK Consistency of Each Comparison Set to the Common Core Standards

Figure 13 shows, for each comparison set, the percent of standards (averaged across raters, see Appendix A) that fell above, at, and under the cognitive demand level of the Common Core standards. In the figure, a line is drawn at 75%, which is the criterion for the number of standards falling above or at the cognitive demand level of the Common Core in order to meet the DOK Consistency statistic. For the Number and Quantity, Algebra, and Statistics and Probability conceptual categories all sets of standards meeting the Categorical Concurrence criterion also meet the DOK criterion. For Geometry, only the Texas College and Career Readiness standards were at or above the cognitive demand of the Common Core standards. For Functions, three of five sets of standards that meet the Categorical Concurrence criterion meet the DOK criterion. These results suggest that for every mathematics conceptual category, the Texas College and Career Readiness standards tend to be similar to or more cognitively complex than the Common Core standards. The California state standards and the Massachusetts state standards are similar to or more cognitively complex than the Common Core standards for four out of five conceptual categories that show categorical concurrence. Figure 13 illustrates these findings. The figure also shows that some standards sets fall just above or just below 75% for some categories and results would be different if an alternative criterion had been selected.

Figure 13. Depth of Knowledge Level Comparisons of the Five Sets of Comparison Standards to Common Core Mathematics Standards



Note: Total percent may equal slightly above or below 100 due to rounding.

Breadth of Coverage

For mathematics, the Breadth of Coverage criterion is related to the number of Common Core clusters within a conceptual category for which a set of comparison standards has matches to the Common Core standards. Strong coverage means a majority of clusters have matches for the conceptual category. Moderate coverage means that more than one cluster has matches for the conceptual category. Limited coverage means that fewer than one cluster or no cluster has matches for the conceptual category.

Coverage in mathematics across the comparison standards was consistently strong, indicating that matching content for each comparison set of standards was dispersed across the clusters of the Common Core standards (see Table 19).

Table 19. Breadth of Coverage Criterion for Mathematics

Common Core mathematics conceptual category	CA	MA	TCCRS	KSUS	IB
Number and Quantity	●	●	●	●	●
Algebra	●	●	●	●	●
Functions	●	●	●	●	●
Geometry	●	●	●	●	●
Statistics and Probability	●	●	●	●	●

CA = California; MA = Massachusetts; TCCRS = Texas College and Career Readiness Standards; KSUS = Knowledge and Skills for University Success; IB = International Baccalaureate.
● means that a majority of clusters in the Common Core conceptual category have at least one matched comparison standard;
● means that more than one cluster in the Common Core conceptual category have at least one matched comparison standard;
● means that one or fewer than one cluster in the Common Core conceptual category have at least one matched comparison standard.

Appendix D presents more information about the distribution of standard matches within each of the five Common Core mathematics conceptual categories.

Chapter 5 | Summary and Discussion

In this chapter we review the purpose of the study, discuss the method chosen to obtain data necessary to answer the three research questions, introduce implications of the findings, and offer suggestions for future research.

The study addresses three research questions:

1. To what extent are the knowledge and skills found in the comparison standards the same as or different from what is described in the Common Core standards? (match)
2. What is the cognitive complexity level of the Common Core standards and to what extent are the matched comparison standards at a higher or lower level of cognitive complexity? (depth)
3. How broadly do the matched comparison standards cover the content of the Common Core standards? (breadth)

The reason for asking these questions is that states are concerned about the rigor of the Common Core standards and the comparison of the Common Core standards to existing standards. Because the Common Core standards sponsors explicitly intended the standards to be at the level of readiness for college and careers, we compared the Common Core standards to five sets of comparison standards that are considered rigorous and, in two cases, are explicitly designated as college and career readiness standards. The two states deemed to have standards that have been recognized as rigorous or otherwise exemplary are California and Massachusetts. The two sets of college and career readiness standards are the Texas College and Career Readiness Standards and the Knowledge and Skills for University Success. Finally, we included an internationally-grounded set of standards from the International Baccalaureate (IB) Diploma Programme.

We measured alignment using three metrics from a modified version of the Webb (1997; 1999; 2002a) method designed to determine standards-to-standards alignment (Cook, 2007; Cook & Wilmes, 2007). The metrics provide a means to quantify and evaluate the extent of overlap between different sets of standards in terms of knowledge, skills, and content. Where overlap exists, the matching standards are then compared to determine relative cognitive demand. The third metric determines breadth of overlap, or coverage, for matching standards. All three metrics are calculated at the highest level of organization for the Common Core (eight strands for English Language Arts (ELA) and literacy; five conceptual categories for mathematics) because the study's goal is to provide an initial high-level view of the relationship between the Common Core standards and other standard systems, not to engage in fine-grained comparisons.

The overall results of the study, as summarized in Table 20, suggest substantial concurrence between the Common Core standards and the comparison standards, with somewhat greater overlap in mathematics than in ELA and literacy. For ELA and literacy, 36 of 40 analyses at the strand level meet the Categorical Concurrence criterion. The "meets" criterion is relatively modest, namely, one or more correspondences between two sets of standards (Cook, 2005). However, for 19 of these comparisons, the mean number of matching standards is above six (six matches is a common criterion for alignment studies that examine concurrence between test items and standards). In places, the number of matches exceeds 20. For mathematics, 25 of 25 analyses at the conceptual category level meet the Categorical Concurrence criterion. For 23 of these comparisons, the mean number of matching standards is above six, in places exceeding 30 matches.

Table 20. Summary of Alignment Findings for Five Sets of Standards Compared to the Common Core Standards

	Common Core content-specific areas	Comparison standards				
		CA	MA	TCCRS	KSUS	IB
English language arts and literacy	Reading for Literature	🔗 ●	🔗 💡 ●	🔗 ●	🔗 💡 ●	🔗 💡 ●
	Reading for Informational Texts	🔗 ●	🔗 💡 ●	🔗 ●	🔗 ●	🔗 ◐
	Writing	🔗 ●	🔗 💡 ●	🔗 ●	🔗 ●	🔗 💡 ●
	Speaking and Listening	🔗 ●	🔗 💡 ●	🔗 ●	🔗 ●	🔗 💡 ●
	Language	🔗 💡 ●	🔗 💡 ●	🔗 💡 ●	🔗 💡 ●	NA ○
	Reading in History/Social Studies	🔗 ●	NA ●	🔗 ●	🔗 ●	🔗 💡 ●
	Reading in Science and Technical Subjects	🔗 💡 ●	NA ◐	🔗 ●	🔗 💡 ●	NA ●
	Writing in History/Social Studies, Science, and Technical Subjects	🔗 💡 ●	🔗 ●	🔗 💡 ●	🔗 ●	🔗 ●
Mathematics	Number & Quantity	🔗 💡 ●	🔗 💡 ●	🔗 💡 ●	🔗 💡 ●	🔗 💡 ●
	Algebra	🔗 💡 ●	🔗 💡 ●	🔗 💡 ●	🔗 💡 ●	🔗 💡 ●
	Functions	🔗 💡 ●	🔗 💡 ●	🔗 💡 ●	🔗 ●	🔗 ●
	Geometry	🔗 ●	🔗 ●	🔗 💡 ●	🔗 ●	🔗 ●
	Statistics & Probability	🔗 💡 ●	🔗 💡 ●	🔗 💡 ●	🔗 💡 ●	🔗 💡 ●

CA = California; MA = Massachusetts; TCCRS = Texas College and Career Readiness Standards; KSUS = Knowledge and Skills for University Success; IB = International Baccalaureate.

Key

- 🔗 Categorical Concurrence met
- 💡 Depth of Knowledge met
- ● Strong coverage
- ◐ Moderate coverage
- ○ Limited or no coverage

In terms of the cognitive demand of the standards, the findings suggest general consistency between the Common Core standards and the five comparison standard sets. There is somewhat greater correspondence in cognitive depth with the mathematics standards comparisons than the ELA and literacy standards comparisons, a finding consistent with other Common Core alignment work (Porter, McMaken, Hwang, & Yang, 2011).

For cognitive complexity in ELA and literacy, 17 of 36 strand-level analyses indicated that the comparison standard sets were at or above the level of the Common Core standards. In 19 cases, the comparison standard sets tended to be rated lower in cognitive complexity than the Common Core standards. This suggests that, as states transition to the Common Core standards, it will be worthwhile to look closely at the level of cognitive complexity in their current standards in relation to the Common Core standards. States should carefully interpret the complexity of the Common Core standards as they implement curriculum frameworks and develop example materials to put the standards into place. The cognitive complexity of the Common Core ELA and literacy standards in particular will be defined by the actual assignments and assessments given to students (see Beach, 2011). States and groups developing curriculum materials for the Common Core standards need to pay close attention to the cognitive challenge level of these materials to ensure they are sufficiently challenging. This is true for many of the strands (although the Language Standards are a notable exception, in part because this area tends to require less cognitive depth), but is particularly important to consider when developing materials to teach and assess reading of informational texts and reading and writing for literacy in the content areas.

For mathematics, 19 of 25 conceptual category-level analyses indicate that the comparison standard sets are at or above the level of the Common Core standards. Conversely, six analyses found comparison standard sets to be below the level of the Common Core standards. The Common Core content areas that tended to be higher are the more conceptually-oriented conceptual category of Geometry, and to a lesser degree, Functions.

Overall, the standards from the comparison sets tend to cover the breadth of topics set forth in the Common Core standards. For ELA and literacy, 37 of 40 strand analyses show strong coverage. Another two strand analyses showed moderate coverage and one showed limited coverage. For mathematics, findings suggest that comparison sets show strong coverage in all 25 conceptual category analyses. While every standard in the Common Core standards may not have a match, the topics around which the Common Core standards are organized are reflected in the comparison standards.

The five sets of comparison standards are not fully adequate to account for the content that might be included in every specific state's standards. Among the comparison standards in this study that are considered rigorous and college and career-preparatory, however, the overall findings suggest a general level of agreement between the Common Core standards and the comparison standards regarding what is important for high school students to know in English language arts and mathematics in order to be highly challenged and ready for college and careers.

The purpose of this study was to ascertain alignment, not to judge the quality of any set of standards in relation to any other set of standards. There is no absolute reason that every set of comparison standards must match the Common Core standards in every area in order to be considered high quality standards. Simply because a set of standards lacks a match to the Common Core standards in one or more areas does not mean the comparison standards are inferior or need to be changed to match the Common Core standards. Knowing something about alignment can help in program planning, but is not necessarily the best or only measure of standards quality.

Closing Note From The Authors

The business of analyzing alignment, cognitive challenge, and coverage between sets of standards remains an approximate art, even when ample care is given to control variation and ensure reliable and valid results. This is one of the reasons we offer findings at a high level of aggregation.

We note that another recent study (Porter et al, 2011) used a different method and reached a different conclusion. This study aligned all grade levels of all state standards to the Common Core using the Survey of Enacted Curriculum (SEC), a method that involves comparing sets of standards to a common framework that intersects particular topics with cognitive demand. Data are converted into proportions and an alignment index calculated to assess commonality in content across the two standard sets. Looking at SEC results across all states, the authors conclude that the Common Core standard represent considerable change from current state standards.

In our opinion, it is possible for both studies to be correct in their conclusions. Our study focuses on the macro level to identify general relationships while the Porter et al. study zeroes in at a much more detailed level. As studies of content alignment move to finer grain size, the probability of achieving universal precise matches begins to decrease. This may be particularly true in the case of the Common Core standards, which were intended to be at a more general level and not to specify each and every element students are expected to learn.

We would also note that in a previous study (Conley, Drummond, de Gonzalez, Rooseboom, & Stout, 2011) we found the Common Core standards to be well aligned with a cross-section of 25 entry-level postsecondary course areas. At the same time, different major or fields of study require different combinations of Common Core standards. In other words, it does not appear that there is a magic set of standards that reflect what every student needs to succeed in every postsecondary setting. The fact that the Common Core standards do not align precisely with each state's existing standards should not be cause for undue alarm if states pay attention to another finding from our previous work (Conley, 2005; Conley et al., 2008) that postsecondary instructors indicate that student thinking skills are as important as any particular bit of content knowledge. While all students need a sound academic foundation, the specific knowledge and skills required for post-high school success will vary. The thinking skills and learning strategies that baccalaureate and career-oriented programs require do appear to be more uniform and consistent. The cognitive challenge level of standards and the ways in which they are taught in classrooms will be as important as the content covered in most cases.

Results from general alignment studies can be informative at a broad level. States still must necessarily engage in a much more fine-grained type of analysis if they want to ascertain the precise relationship between and among each standard in two systems, the existing state standards and the Common Core standards.

Our advice to states is not to lose sight of the forest for the trees. States will be well served to look for patterns along with specific match statistics and address issues of cognitive challenge in particular by the liberal use of example materials and exemplar student work in any standards crosswalk and when the Common Core standards are rolled out statewide. This is true and necessary for both ELA and mathematics. It is perhaps more obvious that the ELA standards will be operationalized not through the literal teaching of the standards statements but through the materials selected to teach the standards at each grade level. As Beach (2011) points out, standards language can be interpreted variably resulting in different translations in different contexts. Educators must make intelligent choices about how students should learn the standards. As we note in the findings section, special attention will need to be paid to informational texts in particular to ensure they reflect a broad range of challenging material and topics.

In mathematics, even though the categorical matches between state standards and the Common Core standards may be very high, it is still worth noting that the cognitive challenge level of these standards will be determined in large measure by the problems students are given to test their knowledge of the many fine-grained and detailed standards. The way the Mathematical Practices standards factor into the Common Core standards poses a challenge to traditional alignment methodology (see Cobb & Jackson, 2011). Although the Mathematical Practices did not lend themselves to inclusion in the current study, they are quite important to keep in mind when considering the intended cognitive challenge level of each of the mathematical standards. The standards authors note the importance of the Mathematical Practice standards and the intent that they be applied to the learning of all of the other mathematics standards. The intention is that in doing so, mathematics will be taught with more robust and complex applications of mathematical processes and principles.

In short, educators should bear in mind the goal of the Common Core standards when considering the mechanics of alignment. That goal is to raise the challenge level for US students, to enable all students to pursue successful futures beyond high school, futures that include additional postsecondary study for most students, and to equip students with a set of core knowledge and skills that enable them to be adaptive learners throughout their lives. Alignment is only a preliminary step in achieving this goal.

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Appendices to
Lining Up: The Relationship between the Common Core State Standards and Five Sets of Comparison Standards

Appendix A | Description of the Depth of Knowledge Calculation

Recall that all raters could match up to three Common Core standards that they judged to be aligned with a comparison standard. The Depth of Knowledge (DOK) Consistency calculation for each Common Core content-specific area uses only raters who had at least one match with a Common Core standard for that content-specific area. (For the purposes of this explanation, we call these “relevant raters.” Those are the raters who identified at least one match in a Common Core content-specific area – specifically a strand for English language arts and literacy and a conceptual category in mathematics.)

For each relevant rater, the WAT first calculates the proportion of comparison standards that are under, at, or above the DOK level of the matched Common Core standard. Then, the statistic involves calculating an overall average for each category (under, at, above) across those relevant raters. If the sum of the at and above categories is over 75%, then the criteria for the DOK Consistency is met.

The Common Core standards below were released in June 2010 and may contain minor differences from the current version due to slight edits after original release. Table B1 contains English language arts and literacy standards and Table B2 contains mathematics standards.

Table B1. English Language Arts and Literacy Common Core Standards Content, with Depth of Knowledge Consensus Ratings

Reference standard number[a]	Standard	DOK rating
1	Reading Standards for Literature	3
1.1	*Key Ideas and Details*	*3*
1.1a	1. Cite strong and thorough textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text, including determining where the text leaves matters uncertain.	3
1.1b	2. Determine two or more themes or central ideas of a text and analyze their development over the course of the text, including how they interact and build on one another to produce a complex account; provide an objective summary of the text.	3
1.1c	3. Analyze the impact of the author's choices regarding how to develop and relate elements of a story or drama (e.g., where a story is set, how the action is ordered, how the characters are introduced and developed).	3
1.2	*Craft and Structure*	*3*
1.2a	4. Determine the meaning of words and phrases as they are used in the text, including figurative and connotative meanings; analyze the impact of specific word choices on meaning and tone, including words with multiple meanings or language that is particularly fresh, engaging, or beautiful. (Include Shakespeare as well as other authors.)	3
1.2b	5. Analyze how an author's choices concerning how to structure specific parts of a text (e.g., the choice of where to begin or end a story, the choice to provide a comedic or tragic resolution) contribute to its overall structure and meaning as well as its aesthetic impact.	3
1.2c	6. Analyze a case in which grasping point of view requires distinguishing what is directly stated in a text from what is really meant (e.g., satire, sarcasm, irony, or understatement).	3
1.3	*Integration of Knowledge and Ideas*	*4*
1.3a	7. Analyze multiple interpretations of a story, drama, or poem (e.g., recorded or live production of a play or recorded novel or poetry), evaluating how each version interprets the source text. (Include at least one play by Shakespeare and one play by an American dramatist.)	4
1.3b	9. Demonstrate knowledge of eighteenth-, nineteenth-, and early-twentieth-century foundational works of American literature, including how two or more texts from the same period treat similar themes or topics.	4
1.4	*Range of Reading and Level of Text Complexity*	*4*
1.4a	10. By the end of grade 11, read and comprehend literature, including stories, dramas, and poems, in the grades 11–CCR text complexity band proficiently, with scaffolding as needed at the high end of the range. By the end of grade 12, read and comprehend literature, including stories, dramas, and poems, at the high end of the grades 11–CCR text complexity band independently and proficiently.	4
2	Reading Standards for Informational Texts	3
2.1	*Key Ideas and Details*	*3*
2.1a	1. Cite strong and thorough textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text, including determining where the text leaves matters uncertain.	3
2.1b	2. Determine two or more central ideas of a text and analyze their development over the course of the text, including how they interact and build on one another to provide a complex analysis; provide an objective summary of the text.	3

[a]The reference standard number refers to the numbering system that was used for the purposes of organizing ratings for this study.

Table B1. continued

Reference standard number[a]	Standard	DOK rating
2.1c	3. Analyze a complex set of ideas or sequence of events and explain how specific individuals, ideas, or events interact and develop over the course of the text.	3
2.2	*Craft and Structure*	*3*
2.2a	4. Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and technical meanings; analyze how an author uses and refines the meaning of a key term or terms over the course of a text (e.g., how Madison defines faction in Federalist No. 10).	3
2.2b	5. Analyze and evaluate the effectiveness of the structure an author uses in his or her exposition or argument, including whether the structure makes points clear, convincing, and engaging.	3
2.2c	6. Determine an author's point of view or purpose in a text in which the rhetoric is particularly effective, analyzing how style and content contribute to the power, persuasiveness, or beauty of the text.	3
2.3	*Integration of Knowledge and Ideas*	*4*
2.3a	7. Integrate and evaluate multiple sources of information presented in different media or formats (e.g., visually, quantitatively) as well as in words in order to address a question or solve a problem.	4
2.3b	8. Delineate and evaluate the reasoning in seminal U.S. texts, including the application of constitutional principles and use of legal reasoning (e.g., in U.S. Supreme Court majority opinions and dissents) and the premises, purposes, and arguments in works of public advocacy (e.g., The Federalist, presidential addresses).	4
2.3c	9. Analyze seventeenth-, eighteenth-, and nineteenth-century foundational U.S. documents of historical and literary significance (including The Declaration of Independence, the Preamble to the Constitution, the Bill of Rights, and Lincoln's Second Inaugural Address) for their themes, purposes, and rhetorical features.	4
2.4	*Range of Reading and Level of Text Complexity*	*4*
2.4a	10. By the end of grade 11, read and comprehend literary nonfiction in the grades 11–CCR text complexity band proficiently, with scaffolding as needed at the high end of the range. By the end of grade 12, read and comprehend literary nonfiction at the high end of the grades 11–CCR text complexity band independently and proficiently.	4
3	Writing Standards	3
3.1	*Text Types and Purposes*	*3*
3.1a	1. Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.	3
3.1b	a. Introduce precise, knowledgeable claim(s), establish the significance of the claim(s), distinguish the claim(s) from alternate or opposing claims, and create an organization that logically sequences claim(s), counterclaims, reasons, and evidence.	3
3.1c	b. Develop claim(s) and counterclaims fairly and thoroughly, supplying the most relevant evidence for each while pointing out the strengths and limitations of both in a manner that anticipates the audience's knowledge level, concerns, values, and possible biases.	4
3.1d	c. Use words, phrases, and clauses as well as varied syntax to link the major sections of the text, create cohesion, and clarify the relationships between claim(s) and reasons, between reasons and evidence, and between claim(s) and counterclaims.	3
3.1e	d. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing.	3
3.1f	e. Provide a concluding statement or section that follows from and supports the argument presented.	2
3.1g	2. Write informative/explanatory texts to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.	3
3.1h	a. Introduce a topic; organize complex ideas, concepts, and information so that each new element builds on that which precedes it to create a unified whole; include formatting (e.g., headings), graphics (e.g., figures, tables), and multimedia when useful to aiding comprehension.	3

Table B1. continued

Reference standard number[a]	Standard	DOK rating
3.1i	b. Develop the topic thoroughly by selecting the most significant and relevant facts, extended definitions, concrete details, quotations, or other information and examples appropriate to the audience's knowledge of the topic.	3
3.1j	c. Use appropriate and varied transitions and syntax to link the major sections of the text, create cohesion, and clarify the relationships among complex ideas and concepts.	3
3.1k	d. Use precise language, domain-specific vocabulary, and techniques such as metaphor, simile, and analogy to manage the complexity of the topic.	3
3.1l	e. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing.	3
3.1m	f. Provide a concluding statement or section that follows from and supports the information or explanation presented (e.g., articulating implications or the significance of the topic).	2
3.1n	3. Write narratives to develop real or imagined experiences or events using effective technique, well-chosen details, and well-structured event sequences.	3
3.1o	a. Engage and orient the reader by setting out a problem, situation, or observation and its significance, establishing one or multiple point(s) of view, and introducing a narrator and/or characters; create a smooth progression of experiences or events.	3
3.1p	b. Use narrative techniques, such as dialogue, pacing, description, reflection, and multiple plot lines, to develop experiences, events, and/or characters.	3
3.1q	c. Use a variety of techniques to sequence events so that they build on one another to create a coherent whole and build toward a particular tone and outcome (e.g., a sense of mystery, suspense, growth, or resolution).	3
3.1r	d. Use precise words and phrases, telling details, and sensory language to convey a vivid picture of the experiences, events, setting, and/or characters.	3
3.1s	e. Provide a conclusion that follows from and reflects on what is experienced, observed, or resolved over the course of the narrative.	3
3.2	*Production and Distribution of Writing*	*3*
3.2a	4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.)	3
3.2b	5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on addressing what is most significant for a specific purpose and audience. (Editing for conventions should demonstrate command of Language standards 1–3, up to and including grades 11–12 on page 54.)	3
3.2c	6. Use technology, including the Internet, to produce, publish, and update individual or shared writing products in response to ongoing feedback, including new arguments or information.	4
3.3	Research to Build and Present Knowledge	4
3.3a	7. Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.	4
3.3b	8. Gather relevant information from multiple authoritative print and digital sources, using advanced searches effectively; assess the strengths and limitations of each source in terms of the task, purpose, and audience; integrate information into the text selectively to maintain the flow of ideas, avoiding plagiarism and overreliance on any one source and following a standard format for citation.	4
3.3c	9. Draw evidence from literary or informational texts to support analysis, reflection, and research.	3
3.3d	a. Apply grades 11–12 Reading standards to literature (e.g., "Demonstrate knowledge of eighteenth-, nineteenth-, and early-twentieth-century foundational works of American literature, including how two or more texts from the same period treat similar themes or topics").	4
3.3e	b. Apply grades 11–12 Reading standards to literary nonfiction (e.g., "Delineate and evaluate the reasoning in seminal U.S. texts, including the application of constitutional principles and use of legal reasoning [e.g., in U.S. Supreme Court Case majority opinions and dissents] and the premises, purposes, and arguments in works of public advocacy [e.g., The Federalist, presidential addresses]").	4

Table B1. continued

Reference standard number[a]	Standard	DOK rating
3.4	*Range of Writing*	*4*
3.4a	10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes, and audiences.	4
4	Speaking and Listening Standards	3
4.1	*Comprehension and Collaboration*	*3*
4.1a	1. Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grades 11-12 topics, texts, and issues, building on others' ideas and expressing their own ideas clearly and persuasively.	3
4.1b	a. Come to discussions prepared, having read and researched material under study; explicitly draw on that preparation by referring to evidence from texts and other research on the topic or issue to stimulate a thoughtful, well-reasoned exchange of ideas.	4
4.1c	b. Work with peers to promote civil, democratic discussions and decision-making, set clear goals and deadlines, and establish individual roles as needed.	3
4.1d	c. Propel conversations by posing and responding to questions that probe reasoning and evidence; ensure a hearing for a full range of positions on a topic or issue; clarify, verify, or challenge ideas and conclusions; and promote divergent and creative perspectives.	3
4.1e	d. Respond thoughtfully to diverse perspectives; synthesize comments, claims, and evidence made on all sides of an issue; resolve contradictions when possible; and determine what additional information or research is required to deepen the investigation or complete the task.	4
4.1f	2. Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.	4
4.1g	3. Evaluate a speaker's point of view, reasoning, and use of evidence and rhetoric, assessing the stance, premises, links among ideas, word choice, points of emphasis, and tone used.	3
4.2	*Presentation of Knowledge and Ideas*	*3*
4.2a	4. Present information, findings, and supporting evidence, conveying a clear and distinct perspective, such that listeners can follow the line of reasoning, alternative or opposing perspectives are addressed, and the organization, development, substance, and style are appropriate to purpose, audience, and a range or formal and informal tasks.	3
4.2b	5. Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.	3
4.2c	6. Adapt speech to a variety of contexts and tasks, demonstrating a command of formal English when indicated or appropriate. (See grades 11–12 Language standards 1 and 3 on page 54 for specific expectations.)	3
5	Language Standards	2
5.1	*Conventions of Standard English*	*1*
5.1a	1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking.	1
5.1b	a. Apply the understanding that usage is a matter of convention, can change over time, and is sometimes contested.	2
5.1c	b. Resolve issues of complex or contested usage, consulting references (e.g., Merriam-Webster's Dictionary of English Usage, Garner's Modern American Usage) as needed.	2
5.1d	2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing.	1
5.1e	a. Observe hyphenation conventions.	1
5.1f	b. Spell correctly.	1
5.2	*Knowledge of Language*	*2*
5.2a	3. Apply knowledge of language to understand how language functions in different contexts, to make effective choices for meaning or style, and to comprehend more fully when reading or listening.	2

Table B1. continued

Reference standard number[a]	Standard	DOK rating
5.2b	a. Vary syntax for effect, consulting references (e.g., Tufte's Artful Sentences) for guidance as needed; apply an understanding of syntax to the study of complex texts when reading.	2
5.3	*Vocabulary Acquisition and Use*	*2*
5.3a	4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grades 11-12 reading and content, choosing flexibly from a range of strategies.	2
5.3b	a. Use context (e.g., the overall meaning of a sentence, paragraph, or text; a word's position or function in a sentence) as a clue to the meaning of a word or phrase.	2
5.3c	b. Identify and correctly use patterns of word changes that indicate different meanings or parts of speech (e.g., conceive, conception, conceivable).	1
5.3d	c. Consult general and specialized reference materials (e.g., dictionaries, glossaries, thesauruses), both print and digital, to find the pronunciation of a word or determine or clarify its precise meaning, its part of speech, its etymology, or its standard usage.	1
5.3e	d. Verify the preliminary determination of the meaning of a word or phrase (e.g., by checking the inferred meaning in context or in a dictionary).	1
5.3f	5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings.	2
5.3g	a. Interpret figures of speech (e.g., hyperbole, paradox) in context and analyze their role in the text.	3
5.3h	b. Analyze nuances in the meaning of words with similar denotations.	2
5.3i	6. Acquire and use accurately general academic and domain-specific words and phrases, sufficient for reading, writing, speaking, and listening at the college- and career-readiness level; demonstrate independence in gathering vocabulary knowledge when considering a word or phrase important to comprehension or expression.	3
6	Reading Standards for Literacy in History/Social Studies	3
6.1	*Key Ideas and Details*	*3*
6.1a	1. Cite specific textual evidence to support analysis of primary and secondary sources, connecting insights gained from specific details to an understanding of the text as a whole.	3
6.1b	2. Determine the central ideas or information of a primary or secondary source; provide an accurate summary that makes clear the relationships among the key details and ideas.	2
6.1c	3. Evaluate various explanations for actions or events and determine which explanation best accords with textual evidence, acknowledging where the text leaves matters uncertain.	3
6.2	*Craft and Structure*	*3*
6.2a	4. Determine the meaning of words and phrases as they are used in a text, including analyzing how an author uses and refines the meaning of a key term over the course of a text (e.g., how Madison defines faction in Federalist No. 10).	2
6.2b	5. Analyze in detail how a complex primary source is structured, including how key sentences, paragraphs, and larger portions of the text contribute to the whole.	3
6.2c	6. Evaluate authors' differing points of view on the same historical event or issue by assessing the authors' claims, reasoning, and evidence.	4
6.3	*Integration of Knowledge and Ideas*	*4*
6.3a	7. Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, as well as in words) in order to address a question or solve a problem.	4
6.3b	8. Evaluate an author's premises, claims, and evidence by corroborating or challenging them with other information.	4
6.3c	9. Integrate information from diverse sources, both primary and secondary, into a coherent understanding of an idea or event, noting discrepancies among sources.	4
6.4	*Range of Reading and Level of Text Complexity*	*3*
6.4a	10. By the end of grade 12, read and comprehend history/social studies texts in the grades 11–12 text complexity band independently and proficiently.	3

Table B1. continued

Reference standard number[a]	Standard	DOK rating
7	Reading Standards for Literacy in Science and Technical Subjects	3
7.1	*Key Ideas and Details*	*3*
7.1a	1. Cite specific textual evidence to support analysis of science and technical texts, attending to important distinctions the author makes and to any gaps or inconsistencies in the account.	3
7.1b	2. Determine the central ideas or conclusions of a text; summarize complex concepts, processes, or information presented in a text by paraphrasing them in simpler but still accurate terms.	2
7.1c	3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.	3
7.2	*Craft and Structure*	*3*
7.2a	4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 11-12 texts and topics.	1
7.2b	5. Analyze how the text structures information or ideas into categories or hierarchies, demonstrating understanding of the information or ideas.	3
7.2c	6. Analyze the author's purpose in providing an explanation, describing a procedure, or discussing an experiment in a text, identifying important issues that remain unresolved.	3
7.3	*Integration of Knowledge and Ideas*	*4*
7.3a	7. Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., quantitative data, video, multimedia) in order to address a question or solve a problem.	4
7.3b	8. Evaluate the hypotheses, data, analysis, and conclusions in a science or technical text, verifying data when possible and corroborating or challenging conclusions with other sources of information.	4
7.3c	9. Synthesize information from a range of sources (e.g., texts, experiments, simulations) into a coherent understanding of a process, phenomenon, or concept, resolving conflicting information when possible.	4
7.4	*Range of Reading and Level of Text Complexity*	*3*
7.4a	10. By the end of Grade 12, read and comprehend science/technical texts in the grades 11–12 text complexity band independently and proficiently.	3
8	Writing Standards for Literacy in History/Social Studies, Science, and Technical Subjects	3
8.1	*Text Types and Purposes*	*3*
8.1a	1. Write arguments focused on discipline-specific content.	3
8.1b	a. Introduce precise, knowledgeable claim(s), establish the significance of the claim(s), distinguish the claim(s) from alternate or opposing claims, and create an organization that logically sequences the claim(s), counterclaims, reasons, and evidence.	3
8.1c	b. Develop claim(s) and counterclaims fairly and thoroughly, supplying the most relevant data and evidence for each while pointing out the strengths and limitations of both claim(s) and counterclaims in a discipline-appropriate form that anticipates the audience's knowledge level, concerns, values, and possible biases.	4
8.1d	c. Use words, phrases, and clauses as well as varied syntax to link the major sections of the text, create cohesion, and clarify the relationships between claim(s) and reasons, between reasons and evidence, and between claim(s) and counterclaims.	3
8.1e	d. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing.	3
8.1f	e. Provide a concluding statement or section that follows from or supports the argument presented.	2
8.1g	2. Write informative/explanatory texts, including the narration of historical events, scientific procedures/experiments, or technical processes.	3
8.1h	a. Introduce a topic and organize complex ideas, concepts, and information so that each new element builds on that which precedes it to create a unified whole; include formatting (e.g., headings), graphics (e.g., figures, tables), and multimedia when useful to aiding comprehension.	3

Table B1. continued

Reference standard number[a]	Standard	DOK rating
8.1i	b. Develop the topic thoroughly by selecting the most significant and relevant facts, extended definitions, concrete details, quotations, or other information and examples appropriate to the audience's knowledge of the topic.	3
8.1j	c. Use varied transitions and sentence structures to link the major sections of the text, create cohesion, and clarify the relationships among complex ideas and concepts.	3
8.1k	d. Use precise language, domain-specific vocabulary and techniques such as metaphor, simile, and analogy to manage the complexity of the topic; convey a knowledgeable stance in a style that responds to the discipline and context as well as to the expertise of likely readers.	3
8.1l	e. Provide a concluding statement or section that follows from and supports the information or explanation provided (e.g., articulating implications or the significance of the topic).	3
8.2	*Production and Distribution of Writing*	*3*
8.2a	4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.	3
8.2b	5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on addressing what is most significant for a specific purpose and audience.	3
8.2c	6. Use technology, including the Internet, to produce, publish, and update individual or shared writing products in response to ongoing feedback, including new arguments or information.	4
8.3	*Research to Build and Present Knowledge*	*4*
8.3a	7. Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.	4
8.3b	8. Gather relevant information from multiple authoritative print and digital sources, using advanced searches effectively; assess the strengths and limitations of each source in terms of the specific task, purpose, and audience; integrate information into the text selectively to maintain the flow of ideas, avoiding plagiarism and overreliance on any one source and following a standard format for citation.	4
8.3c	9. Draw evidence from informational texts to support analysis, reflection, and research.	3
8.4	*Range of Writing*	*4*
8.4a	10. Write routinely over extended time frames (time for reflection and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.	4

[a]The reference standard number refers to the numbering system that was used for the purposes of organizing ratings for this study.

Table B2. Mathematics Common Core Standards Content, with Depth of Knowledge (DOK) Consensus Ratings

Reference standard number[a]	Standard	DOK rating
1	Number and Quantity	1
	The Real Number System	
1.1	*Extend the properties of exponents to rational exponents.*	*2*
1.1a	1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{1/3}$ *to be the cube root of 5 because we want* $(5^{1/3})^3 = 5^{(1/3)3}$ *to hold, so* $(5^{1/3})^3$ *must equal 5.*	2
1.1b	2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.	1
1.2	*Use properties of rational and irrational numbers.*	*3*
1.2a	3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	3
	Quantities	
1.3	*Reason quantitatively and use units to solve problems.*	*2*
1.3a	1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	2
1.3b	2. Define appropriate quantities for the purpose of descriptive modeling.	2
1.3c	3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	2
	The Complex Number System	
1.4	*Perform arithmetic operations with complex numbers.*	*1*
1.4a	1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b being real.	1
1.4b	2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	1
1.4c	3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.	2
1.5	Represent complex numbers and their operations on the complex plane.	2
1.5a	4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.	2
1.5b	5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(1 - \sqrt{3}i)^3 = 8$ *because* $(1 - \sqrt{3}i)$ *has modulus* 2 *and argument* 120°.	2
1.5c	6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.	1
1.6	*Use complex numbers in polynomial identities and equations.*	*2*
1.6a	7. Solve quadratic equations with real coefficients that have complex solutions.	1
1.6b	8. (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.	2
1.6c	9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.	2

[a]The reference standard number refers to the numbering system that was used for the purposes of organizing ratings for this study.

(+) is an indication in the Common Core mathematics standards document to show that some mathematics standards represent advanced content and are intended to prepare students for advanced courses (see Chapter 2 for description).

★is an indication in the Common Core mathematics standards document to show a modeling standard (see Chapter 2 for description).

Table B2. continued

Reference standard number[a]	Standard	DOK rating
	Vector and Matrix Quantities	
1.7	*Represent and model with vector quantities.*	*1*
1.7a	1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., **v**, |***v***|, ||***v***||, *v*).	1
1.7b	2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.	1
1.7c	3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.	2
1.8	*Perform operations on vectors.*	*1*
1.8a	4. (+) Add and subtract vectors.	1
1.8b	a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.	1
1.8c	b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.	2
1.8d	c. Understand vector subtraction ***v – w*** as ***v + (–w)***, where ***–w*** is the additive inverse of ***w***, with the same magnitude as ***w*** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.	2
1.8e	5. (+) Multiply a vector by a scalar.	1
1.8f	a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.	1
1.8g	b. Compute the magnitude of a scalar multiple *c**v*** using ||*c**v***|| = |*c*|*v*. Compute the direction of cv knowing that when |*c*|*v* ≠ 0, the direction of *c**v*** is either along ***v*** (for *c* > 0) or against **v** (for *c* < 0).	1
1.9	*Perform operations on matrices and use matrices in applications.*	*1*
1.9a	6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.	2
1.9b	7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.	1
1.9c	8. (+) Add, subtract, and multiply matrices of appropriate dimensions.	1
1.9d	9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.	1
1.9e	10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.	1
1.9f	11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.	1
1.9g	12. (+) Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.	2
2	Algebra	2
	Seeing Structure in Expressions	
2.1	*Interpret the structure of expressions*	*2*
2.1a	1. Interpret expressions that represent a quantity in terms of its context.★	2
2.1b	a. Interpret parts of an expression, such as terms, factors, and coefficients.	1
2.1c	b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*	2

Table B2. continued

Reference standard number[a]	Standard	DOK rating
2.1d	2. Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.	2
2.2	*Write expressions in equivalent forms to solve problems*	*2*
2.2a	3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.	2
2.2b	a. Factor a quadratic expression to reveal the zeros of the function it defines.	2
2.2c	b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.	2
2.2d	c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*	2
2.2e	4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*	2
	Arithmetic with Polynomials and Rational Functions	
2.3	*Perform arithmetic operations on polynomials*	*1*
2.3a	1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	1
2.4	*Understand the relationship between zeros and factors of polynomials*	*2*
2.4a	2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is p(a), so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	2
2.4b	3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	2
2.5	*Use polynomial identities to solve problems*	*3*
2.5a	4. Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*	3
2.5b	5. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. (The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.)	1
2.6	*Rewrite rational expressions*	*2*
2.6a	6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	2
2.6b	7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	2
	Creating Equations	
2.7	*Create equations that describe numbers or relationships*	*3*
2.7a	1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*	3
2.7b	2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	3
2.7c	3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*	3
2.7d	4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V =IR to highlight resistance R.*	1

Table B2. continued

Reference standard number[a]	Standard	DOK rating
	Reasoning with Equations and Inequalities	
2.8	*Understand solving equations as a process of reasoning and explain the reasoning*	*3*
2.8a	1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	3
2.8b	2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	2
2.9	*Solve equations and inequalities in one variable*	*2*
2.9a	3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	1
2.9b	4. Solve quadratic equations in one variable.	1
2.9c	a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.	2
2.9d	b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	2
2.10	*Solve systems of equations*	*2*
2.10a	5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	3
2.10b	6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	2
2.10c	7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ *and the circle* $x^2 + y^2 = 3$.	2
2.10d	8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.	1
2.10e	9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).	2
2.11	*Represent and solve equations and inequalities graphically*	*2*
2.11a	10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	1
2.11b	11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★	2
2.11c	12. Graph the solutions to a linear inequality in two variables as a halfplane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	2
3	Functions	2
	Interpreting Functions	
3.1	*Understand the concept of a function and use function notation*	*1*
3.1a	1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.	1

Table B2. continued

Reference standard number[a]	Standard	DOK rating
3.1b	2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	2
3.1c	3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.	1
3.2	*Interpret functions that arise in applications in terms of the context*	*2*
3.2a	4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★	2
3.2b	5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★	2
3.2c	6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★	2
3.3	*Analyze functions using different representations*	*2*
3.3a	7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★	2
3.3b	a. Graph linear and quadratic functions and show intercepts, maxima, and minima.	1
3.3c	b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.	2
3.3d	c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.	2
3.3e	d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.	2
3.3f	e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	2
3.3g	8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.	2
3.3h	a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.	2
3.3i	b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*	2
3.3j	9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	2
	Building Functions	
3.4	*Build a function that models a relationship between two quantities*	*3*
3.4a	1. Write a function that describes a relationship between two quantities.★	3
3.4b	a. Determine an explicit expression, a recursive process, or steps for calculation from a context.	3
3.4c	b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*	3

Table B2. continued

Reference standard number[a]	**Standard**	**DOK rating**
3.4d	c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*	3
3.4e	2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★	3
3.5	*Build new functions from existing functions*	*2*
3.5a	3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*	3
3.5b	4. Find inverse functions.	2
3.5c	a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *for* $x > 0$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$.	2
3.5d	b. (+) Verify by composition that one function is the inverse of another.	1
3.5e	c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.	1
3.5f	d. (+) Produce an invertible function from a non-invertible function by restricting the domain.	2
3.5g	5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.	2
	Linear and Exponential Models	
3.6	*Construct and compare linear and exponential models and solve problems*	*2*
3.6a	1. Distinguish between situations that can be modeled with linear functions and with exponential functions.	2
3.6b	a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.	3
3.6c	b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.	1
3.6d	c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	1
3.6e	2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	2
3.6f	3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	2
3.6g	4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.	2
3.7	*Interpret expressions for functions in terms of the situation they model*	*2*
3.7a	5. Interpret the parameters in a linear or exponential function in terms of a context.	2
	Trigonometric Functions	
3.8	*Extend the domain of trigonometric functions using the unit circle*	*3*
3.8a	1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	1
3.8b	2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	3

Table B2. continued

Reference standard number[a]	Standard	DOK rating
3.8c	3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and p/6, and use the unit circle to express the values of sine, cosine, and tangent for *x, π+x*, and *2π–x* in terms of their values for *x*, where *x* is any real number.	2
3.8d	4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	3
3.9	*Model periodic phenomena with trigonometric functions*	*2*
3.9a	5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★	2
3.9b	6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.	1
3.9c	7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★	2
3.10	*Prove and apply trigonometric identities*	*4*
3.10a	8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to calculate trigonometric ratios.	3
3.10b	9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.	4
4	Geometry	3
	Congruence	
4.1	*Experiment with transformations in the plane*	*2*
4.1a	1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	1
4.1b	2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	2
4.1c	3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	2
4.1d	4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	2
4.1e	5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	3
4.2	*Understand congruence in terms of rigid motions*	*3*
4.2a	6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	2
4.2b	7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	3
4.2c	8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	3
4.3	*Prove geometric theorems*	*4*
4.3a	9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; and points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*	4

Table B2. continued

Reference standard number[a]	Standard	DOK rating
4.3b	10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; and the medians of a triangle meet at a point.*	4
4.3c	11. Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*	4
4.4	*Make geometric constructions*	*3*
4.4a	12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Examples include: copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*	3
4.4b	13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	3
	Similarity, Right Triangles, and Trigonometry	
4.5	*Understand similarity in terms of similarity transformations*	*2*
4.5a	1. Verify experimentally the properties of dilations given by a center and a scale factor:	3
4.5b	a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.	1
4.5c	b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	1
4.5d	2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	3
4.5e	3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	2
4.6	*Prove theorems involving similarity*	*4*
4.6a	4. Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*	4
4.6b	5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	4
4.7	*Define trigonometric ratios and solve problems involving right triangles*	*2*
4.7a	6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	1
4.7b	7. Explain and use the relationship between the sine and cosine of complementary angles.	2
4.7c	8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	2
4.8	*Apply trigonometry to general triangles*	*3*
4.8a	9. (+) Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.	3
4.8b	10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.	4
4.8c	11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).	3
	Circles	
4.9	*Understand and apply theorems about circles*	*3*
4.9a	1. Prove that all circles are similar.	3

Table B2. continued

Reference standard number[a]	Standard	DOK rating
4.9b	2. Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; and the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*	3
4.9c	3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	3
4.9d	4. (+) Construct a tangent line from a point outside a given circle to the circle.	2
4.10	*Find arc lengths and areas of sectors of circles*	*3*
4.10a	5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	3
	Expressing Geometric Properties with Equations	
4.11	*Translate between the geometric description and the equation for a conic section*	*2*
4.11a	1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	2
4.11b	2. Derive the equation of a parabola given a focus and directrix.	2
4.11c	3. (+) Derive the equations of ellipses and hyperbolas given foci and directrices.	2
4.12	*Use coordinates to prove simple geometric theorems algebraically*	*3*
4.12a	4. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).*	3
4.12b	5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	3
4.12c	6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	2
4.12d	7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★	2
	Geometric Measurement and Dimension	
4.13	*Explain volume formulas and use them to solve problems*	*3*
4.13a	1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*	3
4.13b	2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.	3
4.13c	3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★	2
4.14	*Visualize relationships between two-dimensional and three-dimensional objects*	*2*
4.14a	4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	2
	Modeling with Geometry	
4.15	*Apply geometric concepts in modeling situations*	*2*
4.15a	1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★	2
4.15b	2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★	2
4.15c	3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★	4

Table B2. continued

Reference standard number[a]	Standard	DOK rating
5	Statistics and Probability★	2
	Interpreting Categorical and Quantitative Data	
5.1	*Summarize, represent, and interpret data on a single count or measurement variable*	*2*
5.1a	1. Represent data with plots on the real number line (dot plots, histograms, and box plots).★	1
5.1b	2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.★	2
5.1c	3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).★	2
5.1d	4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.★	2
5.2	*Summarize, represent, and interpret data on two categorical and quantitative variables*	*2*
5.2a	5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.★	2
5.2b	6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★	2
5.2c	a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.	2
5.2d	b. Informally assess the fit of a function by plotting and analyzing residuals.	3
5.2e	c. Fit a linear function for a scatter plot that suggests a linear association.	1
5.3	*Interpret linear models*	*2*
5.3a	7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.★	2
5.3b	8. Compute (using technology) and interpret the correlation coefficient of a linear fit.★	2
5.3c	9. Distinguish between correlation and causation.★	2
	Making Inferences and Justifying Conclusions	
5.4	*Understand and evaluate random processes underlying statistical experiments*	*3*
5.4a	1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.★	1
5.4b	2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*★	3
5.5	*Make inferences and justify conclusions from sample surveys, experiments, and observational studies*	*3*
5.5a	3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.★	2
5.5b	4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.★	2
5.5c	5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.★	3
5.5d	6. Evaluate reports based on data.★	3

Table B2. continued

Reference standard number[a]	Standard	DOK rating
	Conditional Probability and the Rules of Probability	
5.6	*Understand independence and conditional probability and use them to interpret data*	*2*
5.6a	1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").★	2
5.6b	2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.★	2
5.6c	3. Understand the conditional probability of *A* given *B* as *P(A* and *B)/P(B)*, and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.★	2
5.6d	4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among mathematics, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*★	2
5.6e	5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*★	2
5.7	*Use the rules of probability to compute probabilities of compound events in a uniform probability model*	*2*
5.7a	6. Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.★	2
5.7b	7. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.★	2
5.7c	8. (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B) P(A|B), and interpret the answer in terms of the model.★	2
5.7d	9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.★	2
	Using Probability to Make Decisions	
5.8	*Calculate expected values and use them to solve problems*	*2*
5.8a	1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.★	2
5.8b	2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.★	2
5.8c	3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*★	2
5.8d	4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*★	2
5.9	*Use probability to evaluate outcomes of decisions*	*2*
5.9a	5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.★	2

Table B2. continued

Reference standard number[a]	Standard	DOK
5.9b	a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*	2
5.9c	b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*	2
5.9d	6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).★	2
5.9e	7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).★	4

[a]The reference standard number refers to the numbering system that was used for the purposes of organizing ratings for this study.

(+) is an indication in the Common Core mathematics standards document to show that some mathematics standards represent advanced content and are intended to prepare students for advanced courses (see Chapter 2 for description).

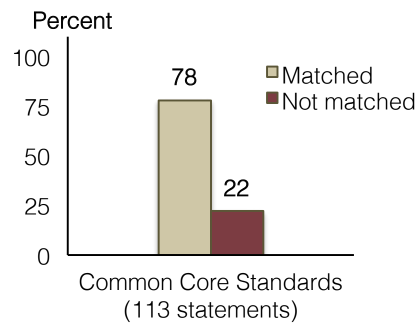
★is an indication in the Common Core mathematics standards document to show a modeling standard (see Chapter 2 for description).

In the sections that follow, we present the alignment findings between each set of individual comparison standards and the Common Core standards. These include (1) the state standards for California; (2) the state standards for Massachusetts; (3) the Texas Career and College Readiness Standards; (4) the Knowledge and Skills for University Success; and 5) the International Baccalaureate Diploma Programme standards. Chapters 3 and 4 presented basic findings. The text in this appendix summarizes additional findings and the Technical Supplement contains even more detailed information on the statistics and raw data.

California Standards

The 11th–12th-grade English language arts standards and the 8th–12th-grade standards in mathematics of the California Content Standards were compared to the Common Core standards.

Figure C1. Percentage of Common Core Standards that Match California Standards for ELA and Literacy



English Language Arts and Literacy Findings for the California Standards

Match

Recall that raters identified up to three Common Core standards as matching each California standard. Figure C1 shows the percent of Common Core statements (n = 113) that any of the nine ELA raters matched to one or more of the California standards (n = 108). The figure collapses findings across all eight English language arts (ELA) and literacy strands. In Figure C2, we divide findings by strand. As the bar graphs indicate, the California standards have matches for 100% of the Common Core standards in three strands, Reading for Literature, Reading for Informational Text, and Reading Standards in History/Social Studies. The California standards have matches to 76-89% of the Common Core standards in the Writing, Speaking and Listening, and Language strands. They have matches to 40% of the Common Core standards in the Reading Standards in Science and Technical Subjects strand and 47% in the Writing Standards in History/Social Studies, Science, and Technical Subjects strand. The data presented here is based on at least one rater matching at least one standard, however the number of matches for any given standard varies widely. Chapter 3 provides the mean and standard deviation for the number of comparison standards matching Common Core standards in each strand. The Technical Supplement displays the exact standards from the Common Core and the comparison sets that were matched.

Every strand meets the criterion for the Categorical Concurrence statistic, which specifies that, across raters, an average of one correspondence in a content area is sufficient.

Figure C2. Percentage of Common Core Standards that Match California Standards for ELA and Literacy Strand

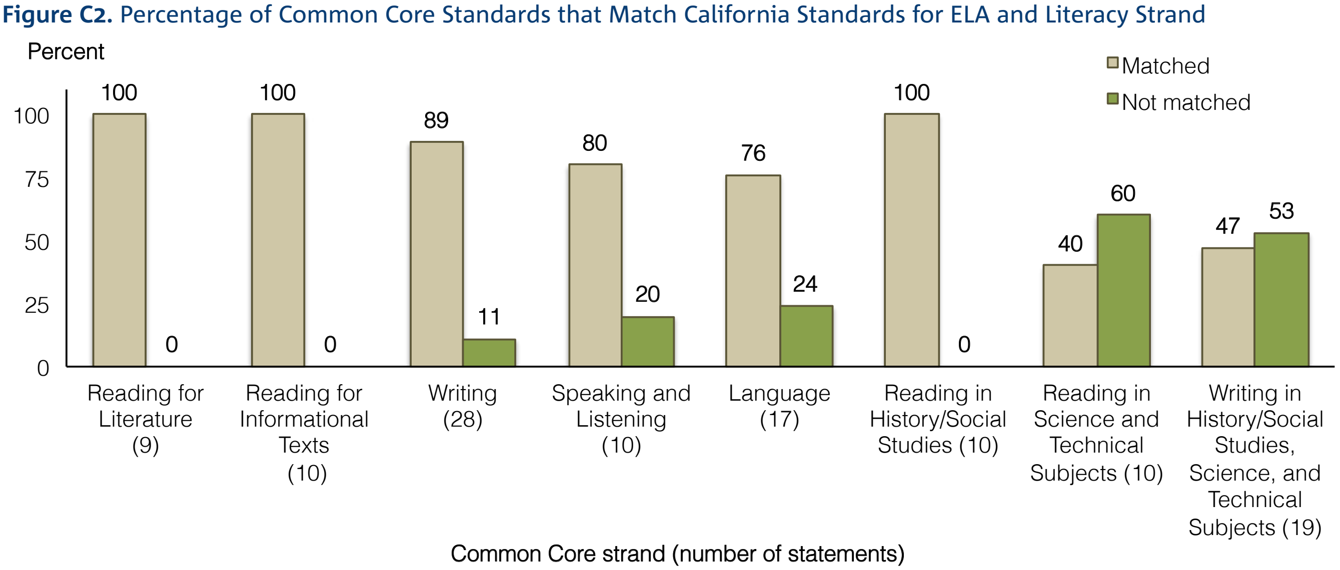


Table C1. Summary for Alignment Between the California Standards and Common Core Standards for English Language Arts and Literacy

Common Core ELA and literacy strand	Categorical Concurrence	Coverage
Reading for Literature	●	●
Reading for Informational Texts	●	●
Writing	●	●
Speaking and Listening	●	●
Language	●	●
Reading in History/Social Studies	●	●
Reading in Science and Technical Subjects	●	●
Writing in History/Social Studies, Science, and Technical Subjects	●	●

Meets the criterion for Categorical Concurrence (averaged across raters, a single correspondence exists between the two sets of standards);
Does not meet the criterion for Categorical Concurrence;
means that a majority of topics in the Common Core category have at least one matched comparison standard;
means that in more than one topic in the Common Core category have at least one matched comparison standard;
means that in one or less than one topic in the Common Core category have at least one matched comparison standard.

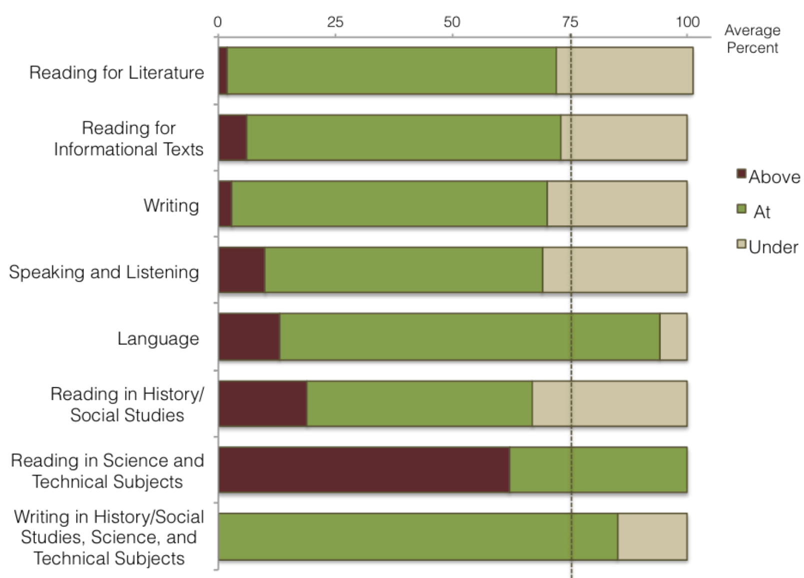
Breadth of Coverage

Breadth of Coverage is strong for all the ELA and literacy strands (the majority of Common Core topics have matching comparison standards within them). This means that the California standards that match to Common Core standards are distributed across content topics within each Common Core strand. Table C1 presents the alignment summary for the Webb and Cook alignment statistics (Cook, 2005; Cook & Wilmes, 2007; Webb, 2002a).

Depth of Knowledge

Across all matched standards for ELA and literacy, an average of 66% (*SD* = 43) of California standards are at the same Depth of Knowledge (DOK) level as the Common Core standards, with an average of 26% (*SD* = 40) of the California standards rated lower in cognitive demand and an average of 8% (*SD* = 24) rated higher. Three of the eight strands (shown in Figure C3) meet the DOK Consistency criterion, which is based on at least 75% of the matching comparison standards being at or above the DOK of the corresponding Common Core standard. As Figure C3 shows, some strands fall close to 75% and results would be different if an alternative criterion had been selected.

Figure C3. California Standards Depth of Knowledge Levels Under, At, and Above the Common Core for ELA and Literacy



Note: Total percent may equal slightly above or below 100 due to rounding.

Mathematics Findings for the California Standards

Match

Figure C4 shows the percent of Common Core statements (*n* = 192) that any of the seven mathematics raters matched to one or more of the California standards (*n* = 185). The figure collapses findings across the five conceptual categories. In Figure C5, we divide findings by conceptual category. As the bar graphs indicate, a high percent of the Common Core standards match California standards in each conceptual category (ranging from 82–91%). Again, the data presented here is based on at least one rater matching at least one standard, however the number of matches for any given standard varies widely. Chapter 4 provides the mean and standard deviation for the number of comparison standards matching Common Core standards in each conceptual category. The Technical Supplement displays the exact standards from the Common Core and the comparison sets that were matched. Every conceptual category meets the criterion for the Categorical Concurrence statistic.

Figure C4. Percentage of Common Core Standards that Match California Standards, for Mathematics

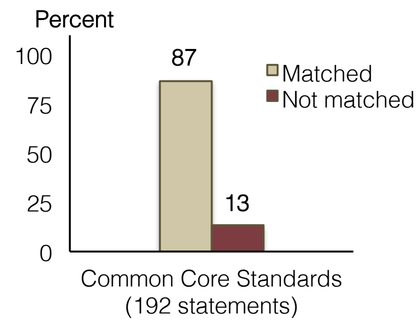


Figure C5. Percentage of Common Core Standards that Match California Standards, by Mathematics Conceptual Category

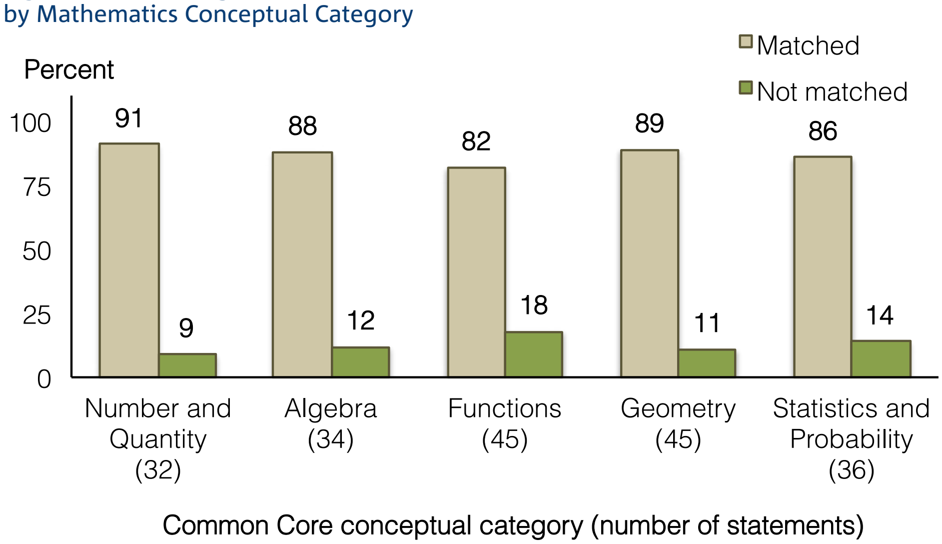


Table C2. Summary for Alignment Between the California Standards and Common Core Standards for Mathematics

Common Core mathematics conceptual category	Categorical Concurrence	Coverage
Number and Quantity		
Algebra		
Functions		
Geometry		
Statistics and Probability		

Meets the criterion for Categorical Concurrence (averaged across raters, a single correspondence exists between the two sets of standards);
Does not meet the criterion for Categorical Concurrence;
means that a majority of topics in the Common Core category have at least one matched comparison standard;
means that in more than one topic in the Common Core category have at least one matched comparison standard;
means that Common Core standards in one or less than one topic in the strand have at least one matched comparison standard.

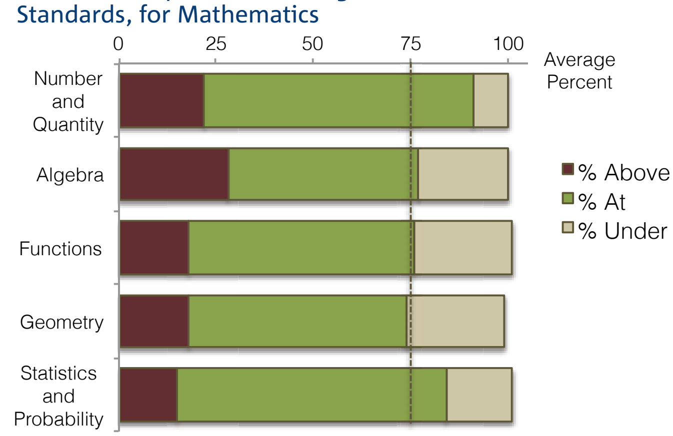
Breadth of Coverage

Breadth of Coverage is strong for all the mathematics conceptual categories (the majority of Common Core clusters have matching comparison standards within them). Table C2 presents the summarized alignment results for mathematics.

Depth of Knowledge

For California mathematics standards that match to Common Core standards, an average of 59% (*SD* = 46) fall at the same level as the Common Core standards with an average of 21% (*SD* = 39) lower in cognitive demand and 20% (*SD* = 38) higher in cognitive demand. Four of the five Common Core conceptual categories (shown in Figure C6) meet the DOK Consistency criterion, with Geometry being the one category that does not meet the criterion. As Figure C6 shows, some categories fall close to 75% and results would be different if an alternative criterion had been selected.

Figure C6. Percentage of California Standards Under, At, or Above the Depth of Knowledge Level of the Common Core Standards, for Mathematics



Note: Total percent may equal slightly above or below 100 due to rounding.

Massachusetts Standards

The study compared the 11th–12th-grade standards from the Massachusetts Curriculum Frameworks to the Common Core standards.

English Language Arts and Literacy Findings for the Massachusetts Standards

Match

Recall that raters identified up to three Common Core standards that correspond with each Massachusetts standard. Figure C7 shows the percent of Common Core statements (n = 113) that any of the nine ELA raters matched to one or more of the Massachusetts standards (n = 41). The figure collapses findings across all eight English language arts (ELA) and literacy strands. In Figure C8, we divide findings by strand. As the bar graphs indicate, the Massachusetts standards have matches for 100% of the Common Core standards in Reading for Literature, and Speaking and Listening. The Massachusetts standards have matches to 71–94% of the Common Core standards in the Reading for Informational Text, Writing, and Language strands. For the Reading Standards in History/Social Studies strand, Reading Standards in Science and Technical Subjects strand, and Writing Standards in History/Social Studies, Science, and Technical Subjects strand, the Massachusetts standards have matches to 20–40% of the Common Core standards.[1] Six of eight strands meet the criterion for the Categorical Concurrence statistic, which specifies that, across raters, an average of one correspondence in a content area is sufficient. The two Reading for Literacy (in History/Social Studies and in Science and Technical Subjects) do not meet the criterion.

[1]The data presented here is based on at least one rater matching at least one standard, however the number of matches for any given standard varies widely. Chapter 3 provides the mean and standard deviation for the number of comparison standards matching Common Core standards in each strand. The Technical Supplement displays the exact standards from the Common Core and the comparison sets that were matched.

Figure C7. Percentage of Common Core Standards that Match Massachusetts Standards, for ELA and Literacy

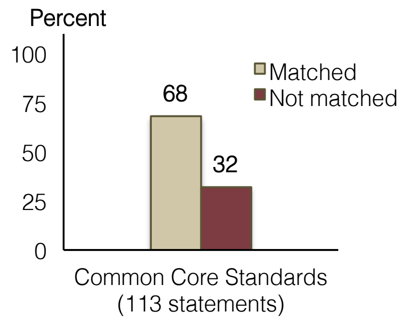


Figure C8. Percentage of Common Core Standards that Match Massachusetts Standards, by ELA and Literacy Strand

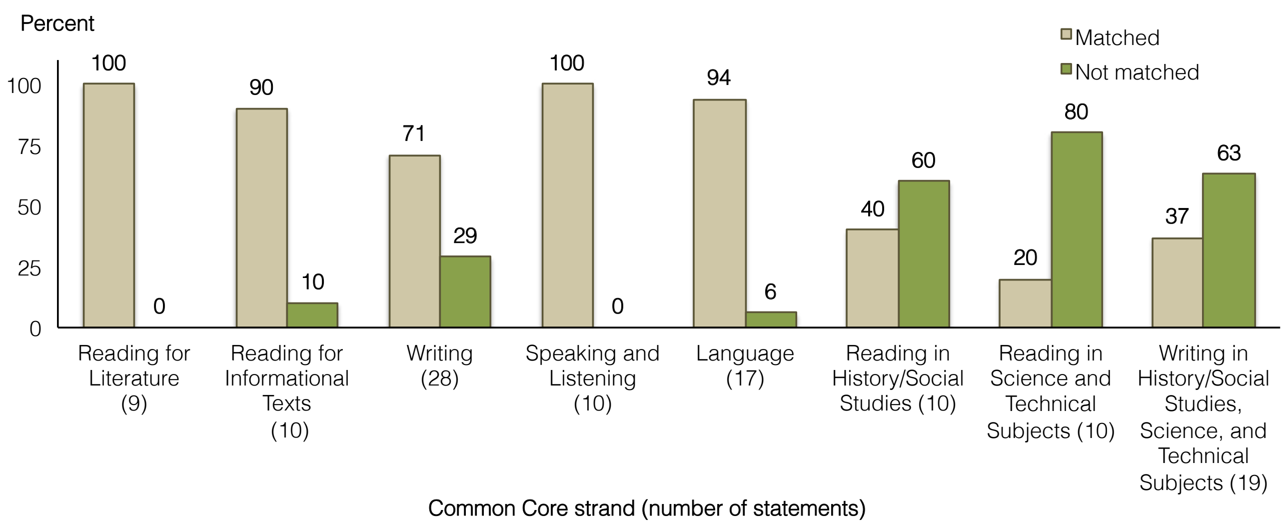


Table C3. Summary for Alignment Between the Massachusetts Standards and the Common Core Standards for English Language Arts and Literacy

Common Core ELA and literacy strand	Categorical Concurrence	Coverage
Reading for Literature	🟢	●
Reading for Informational Texts	🟢	●
Writing	🟢	●
Speaking and Listening	🟢	●
Language	🟢	●
Reading in History/Social Studies	🔴	●
Reading in Science and Technical Subjects	🔴	◐
Writing in History/Social Studies, Science, and Technical Subjects	🟢	●

🟢 Meets the criterion for Categorical Concurrence (averaged across raters, a single correspondence exists between the two sets of standards);
🔴 Does not meet the criterion for Categorical Concurrence;
● means that a majority of topics in the Common Core category have at least one matched comparison standard;
◐ means that in more than one topic in the Common Core category have at least one matched comparison standard;
○ means that in one or less than one topic in the Common Core category have at least one matched comparison standard.

Breadth of Coverage

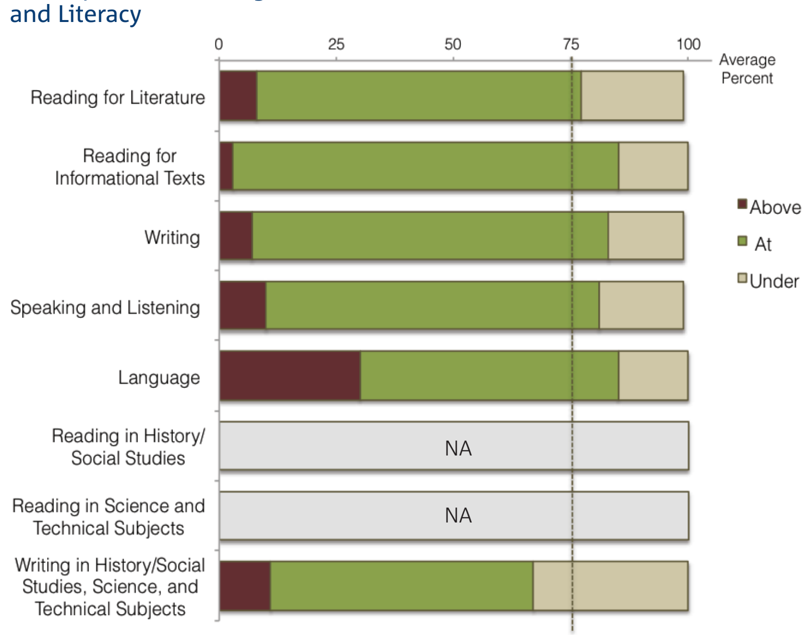
Breadth of Coverage is strong for seven strands (for these, the majority of Common Core topics have matching comparison standards within them). The Reading Standards in History/Social Studies strand is moderate in terms of coverage. This means that Common Core standards match to Massachusetts standards in more than one topic but not a majority of topics within the strand. A summary table (Table C3) presents the ELA and literacy alignment results for the Webb and Cook alignment statistics (Cook, 2005; Cook & Wilmes, 2007; Webb, 2002a).

Depth of Knowledge

Across all matched standards for ELA and literacy, an average of 69% (*SD* = 45) of Massachusetts standards are at the same Depth of Knowledge (DOK) level as the Common Core standards. According to study ratings, an average of 18% (*SD* = 38) are less cognitively demanding and an average of 13% (*SD* = 32) are more cognitively demanding.

Of the six strands meeting the Categorical Concurrence criterion, five meet the DOK Consistency criterion, which is based on at least 75% of the matching Massachusetts standards being at or above the DOK of the corresponding Common Core standard (see Figure C9). One strand did not meet the criterion: Writing Standards for Literacy in History/Social Studies, Science, and Technical Subjects. As Figure C9 shows, some strands fall close to 75% and results would be different if an alternative criterion had been selected.

Figure C9. Percentage of Massachusetts Standards Under, At, or Above the Depth of Knowledge Level of the Common Core Standards, for ELA and Literacy



Note: Total percent may equal slightly above or below 100 due to rounding.

Mathematics Findings for the Massachusetts Standards

Match

Figure C10 shows the percent of Common Core statements (n = 192) that have matches in the Massachusetts standards (n = 108). The figure collapses findings across the five conceptual categories. In Figure C11, we divide findings by conceptual category. As the bar graphs indicate, a high percent of the Common Core standards cover the Massachusetts standards in each conceptual category (ranging from 72–100%).[2] Every conceptual category meets the criterion for the Categorical Concurrence statistic.

[2]The data presented here is based on at least one rater matching at least one standard, however the number of matches for any given standard varies widely. Chapter 4 provides the mean and standard deviation for the number of comparison standards matching Common Core standards in each conceptual category. The Technical Supplement displays the exact standards from the Common Core and the comparison sets that were matched.

Figure C10. Percentage of Common Core Standards that Match Massachusetts Standards, for Mathematics

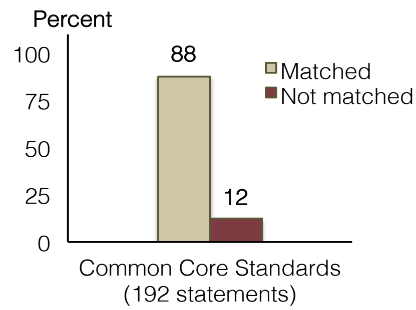


Figure C11. Percentage of Common Core Standards that Match Massachusetts Standards, by Mathematics Conceptual Category

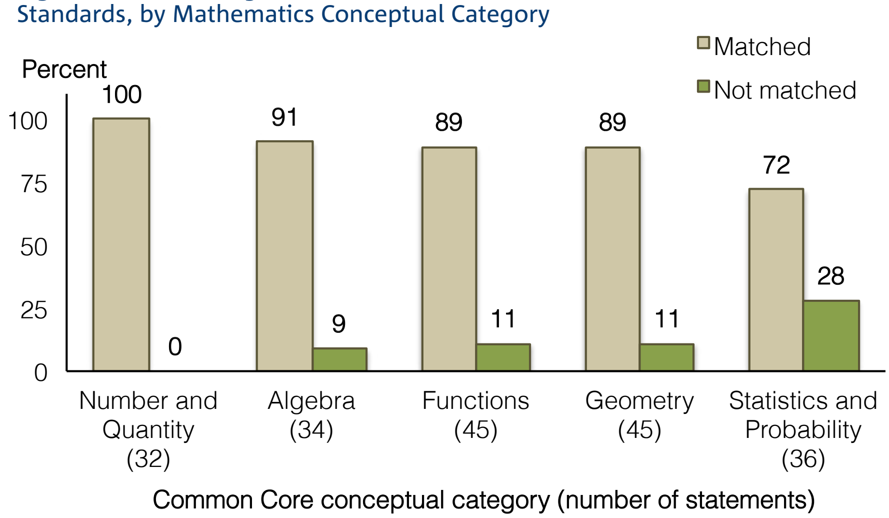


Table C4. Summary for Alignment Between the Massachusetts Standards and Common Core Standards in Mathematics

Common Core mathematics conceptual category	Categorical Concurrence	Coverage
Number and Quantity		
Algebra		
Functions		
Geometry		
Statistics and Probability		

Meets the criterion for Categorical Concurrence (averaged across raters, a single correspondence exists between the two sets of standards);
Does not meet the criterion for Categorical Concurrence;
means that a majority of topics in the Common Core category have at least one matched comparison standard;
means that in more than one topic in the Common Core category have at least one matched comparison standard;
means that in one or less than one topic in the Common Core category have at least one matched comparison standard.

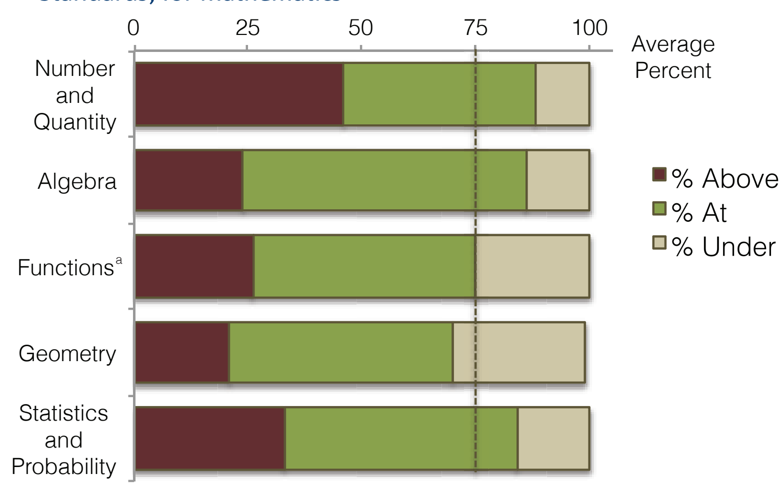
Breadth of Coverage

Breadth of Coverage is strong for all the mathematics conceptual categories (the majority of Common Core clusters have matching comparison standards within them). Table C4 presents the summarized alignment results for mathematics.

Depth of Knowledge

For the Massachusetts standards that match to Common Core standards, an average of 51% (*SD* = 47) fall at the same level of cognitive demand as the Common Core standards. An average of 20% (*SD* = 38) of the Massachusetts standards are lower in cognitive demand than the Common Core standards, and an average of 28% (*SD* = 43) are rated above the Common Core standards. Four of the five Common Core conceptual categories meet the DOK Consistency criterion (see Figure C12). According to ratings, the Geometry category did not meet the criterion. As Figure C12 shows, some categories fall close to or at 75% and results would be different if an alternative criterion had been selected.

Figure C12. Percentage of Massachusetts Standards Under, At, or Above the Depth of Knowledge Level of the Common Core Standards, for Mathematics



Note: Total percent may equal slightly above or below 100 due to rounding.
[a]The Functions conceptual category meets the DOK criterion, but due to rounding, the value appears at exactly 75%.

Texas College and Career Readiness Standards

The study compared the English language arts, mathematics, and cross-disciplinary standards from the Texas College and Career Readiness Standards (TCCRS) to the Common Core standards.

English Language Arts Findings for the TCCRS Standards

Match

Raters identified up to three Common Core standards with each TCCRS standard. Figure C13 shows the percent of Common Core statements (n = 113) that any of the nine ELA raters matched to one or more of the TCCRS standards (n = 89). The figure collapses findings across all eight English language arts (ELA) and literacy strands. In Figure C14, we divide findings by strand. As the bar graphs indicate, in five strands the TCCRS standards have matches to 100% of the Common Core standards. Two additional strands have matches to 88–90% of the Common Core standards (Language and Reading Standards in History/Social Studies). The TCCRS standards have matches to 68% of the Common Core standards in the Writing Standards in History/Social Studies, Science, and Technical Subjects strand.[3] Every strand meets the criterion for the Categorical Concurrence statistic, which specifies that, across raters, an average of one correspondence in a content area is sufficient.

[3]The data presented here is based on at least one rater matching at least one standard, however the number of matches for any given standard varies widely. Chapter 3 provides the mean and standard deviation for the number of comparison standards matching Common Core standards in each strand. The Technical Supplement displays the exact standards from the Common Core and the comparison sets that were matched.

Figure C13. Percentage of Common Core Standards that Match Texas College and Career Readiness Standards, for ELA and Literacy

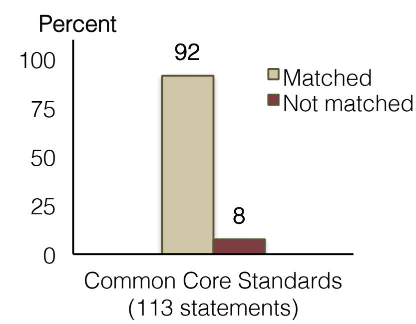


Figure C14. Percentage of Common Core Standards that Match Texas College and Career Readiness Standards, by ELA and Literacy Strand

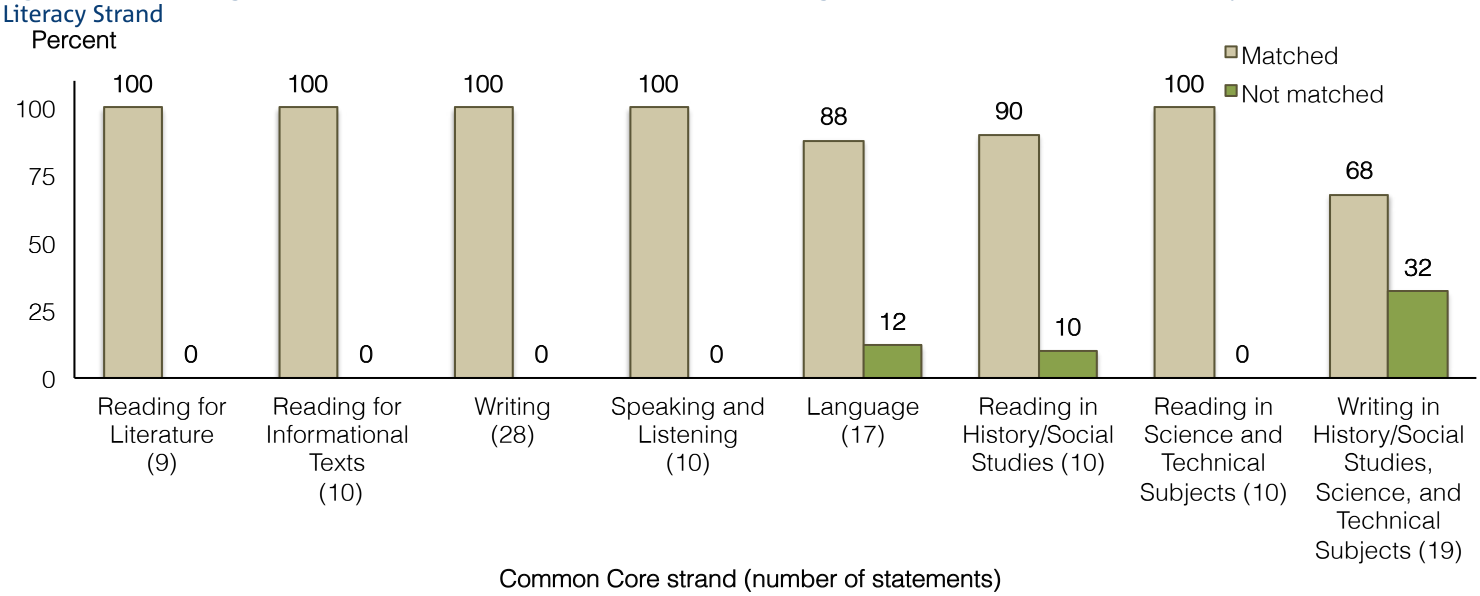


Table C5. Summary of Alignment Between the Texas College and Career Readiness Standards and Common Core for English Language Arts and Literacy

Common Core ELA and literacy strand	Categorical Concurrence	Coverage
Reading for Literature	●	●
Reading for Informational Texts	●	●
Writing	●	●
Speaking and Listening	●	●
Language	●	●
Reading in History/Social Studies	●	●
Reading in Science and Technical Subjects	●	●
Writing in History/Social Studies, Science, and Technical Subjects	●	●

● Meets the criterion for Categorical Concurrence (averaged across raters, a single correspondence exists between the two sets of standards);
● Does not meet the criterion for Categorical Concurrence;
● means that in a majority of topics in the Common Core category have at least one matched comparison standard;
◐ means that in more than one topic in the Common Core category have at least one matched comparison standard;
○ means that in one or less than one topic in the Common Core category have at least one matched comparison standard.

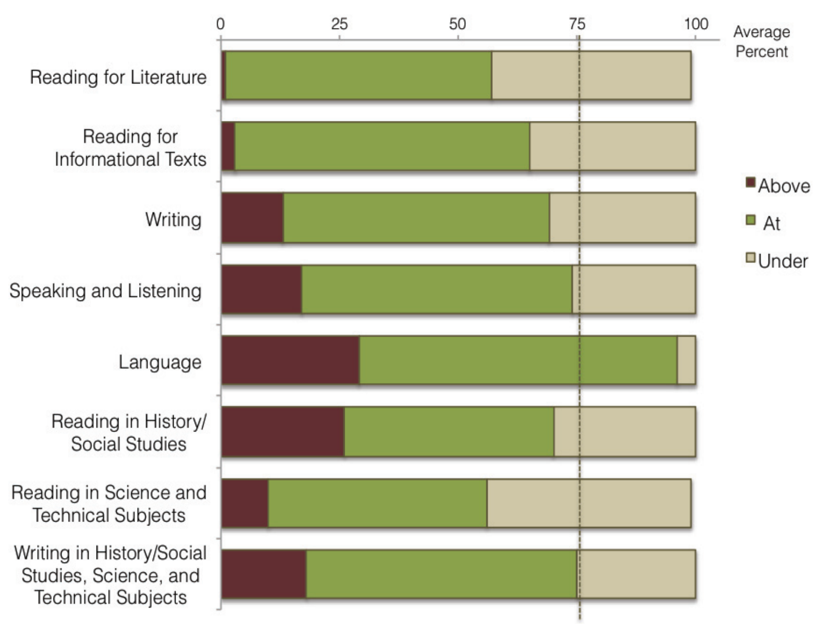
Breadth of Coverage

Breadth of Coverage is rated as strong for all ELA and literacy strands (the majority of Common Core topics have matching comparison standards within them). This means that the Massachusetts standards that match to Common Core standards are distributed across content topics within each Common Core strand. Table C5 presents the alignment summary for the Webb and Cook statistics (Cook, 2005; Cook & Wilmes, 2007; Webb, 2002a).

Depth of Knowledge

Across all matched standards for ELA and literacy, an average of 57% (*SD* = 46) are at the same level as the Common Core standards. According to ratings, an average of 29% (*SD* = 43) of the standards are lower, and 14% (*SD* = 33) are higher in cognitive demand. The Depth of Knowledge (DOK) Consistency criterion is met for two of eight strands: the Language Standards and the Writing Standards in History/Social Studies, Science, and Technical Subjects. For the other six Common Core strands, the criterion is not met. Figure C15 displays the results graphically. As the figure shows, some strands fall close to 75% and results would be different if an alternative criterion had been selected.

Figure C15. Percent of Texas College and Career Readiness Standards Under, At, or Above the Depth of Knowledge Level of the Common Core Standards, for ELA and Literacy



Note: Total percent may equal slightly above or below 100 due to rounding.

Mathematics Findings for the TCCRS Standards

Match

Figure C16 shows the percent of Common Core statements (n = 192) that have matches in the TCCRS mathematics standards (n = 115). The figure collapses findings across the five conceptual categories. In Figure C17, we divide findings by conceptual category. As the bar graphs indicate, the TCCRS standards have matches to a high percent (ranging from 76–94%) of the Common Core standards in four of five conceptual categories. For Number and Quantity, the TCCRS standards have matches to only 34% of the Common Core standards.[4] Every conceptual category meets the criterion for the Categorical Concurrence statistic.

[4]The data presented here is based on at least one rater matching at least one standard, however the number of matches for any given standard varies widely. Chapter 4 provides the mean and standard deviation for the number of comparison standards matching Common Core standards in each conceptual category. The Technical Supplement displays the exact standards from the Common Core and the comparison sets that were matched.

Figure C16. Percentage of Common Core Standards that Match Texas College and Career Readiness Standards, for Mathematics

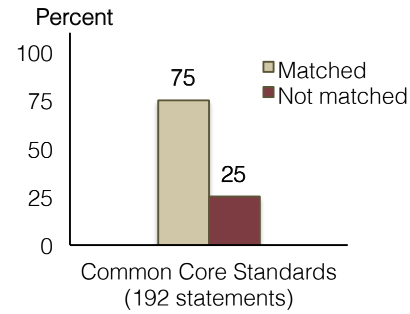


Figure C17. Percentage of Common Core Standards that Match Texas College and Career Readiness Standards, by Mathematics Conceptual Category

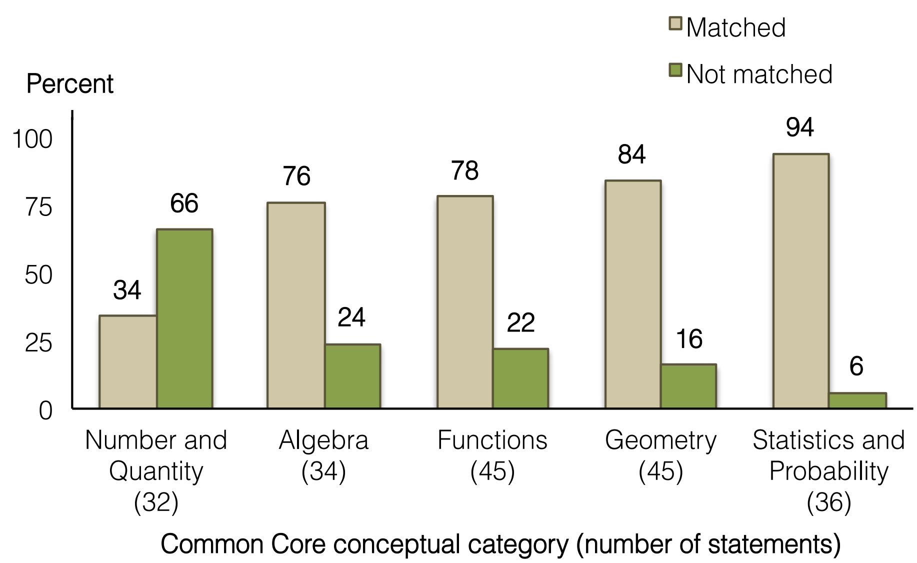


Table C6. Summary for Alignment Between the Texas College and Career Readiness Standards and Common Core Standards in Mathematics

Common Core mathematics conceptual category	Categorical Concurrence	Coverage
Number and Quantity		
Algebra		
Functions		
Geometry		
Statistics and Probability		

Meets the criterion for Categorical Concurrence (averaged across raters, a single correspondence exists between the two sets of standards);
Does not meet the criterion for Categorical Concurrence;
means that in a majority of topics in the Common Core category have at least one matched comparison standard; means that in more than one topic in the Common Core category have at least one matched comparison standard; means that in one or less than one topic in the Common Core category have at least one matched comparison standard.

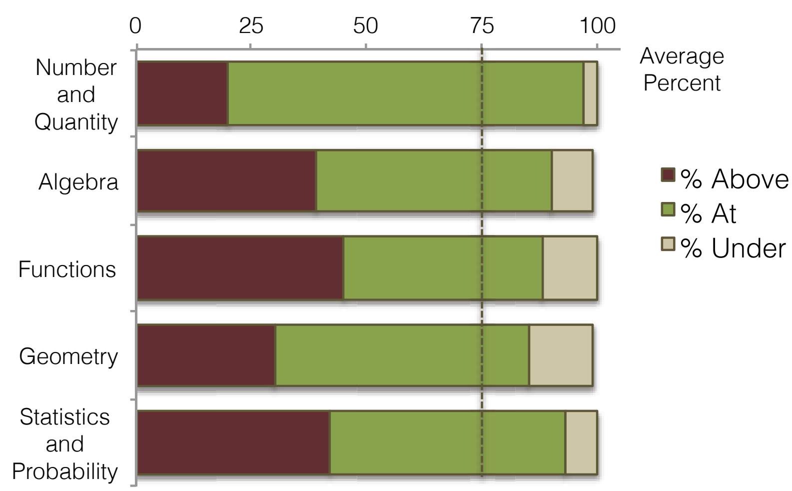
Breadth of Coverage

Breadth of Coverage is strong for all mathematics conceptual categories, indicating that the matching standards are distributed across a majority of Common Core mathematics clusters within each conceptual category. Table C6 summarizes the findings.

Depth of Knowledge

Across all matched standards in mathematics, an average of 53% (*SD* = 46) fall at the same level as the Common Core standards, with an average of 10% (*SD* = 28) of the Texas CCRS standards rated lower in cognitive demand and an average of 37% (*SD* = 45) rated higher. All five conceptual categories meet the DOK Consistency criterion.

Figure C18. Percent of Texas College and Career Readiness Standards Under, At, or Above the Depth of Knowledge Level of the Common Core Standards, for Mathematics



Note: Total percent may equal slightly above or below 100 due to rounding.

Knowledge and Skills for University Success

English standards and mathematics standards sections from the Knowledge and Skills for University Success (KSUS) Standards were compared to the Common Core standards.

English Language Arts and Literacy Findings for the KSUS Standards

Match

Figure C19 shows the percent of Common Core statements (*n* = 113) that any of the nine ELA raters matched to one or more of the KSUS standards (*n* = 73). The figure collapses findings across all eight English language arts (ELA) and literacy strands. In Figure C20, we divide findings by strand. As the bar graphs indicate, the KSUS standards have matches for 100% of the Common Core standards for the Reading for Literature strand. The KSUS standards have matches to 79–96% of the Common Core standards for six strands. They have matches to 60% of the Common Core standards in the Reading Standards in Science and Technical Subjects strand.[5] Every strand meets the criterion for the Categorical Concurrence statistic, which specifies that, across raters, an average of one correspondence in a content area is sufficient.

[5]The data presented here is based on at least one rater matching at least one standard, however the number of matches for any given standard varies widely. Chapter 3 provides the mean and standard deviation for the number of comparison standards matching Common Core standards in each strand. The Technical Supplement displays the exact standards from the Common Core and the comparison sets that were matched.

Figure C19. Percentage of Common Core Standards that Match Knowledge and Skills for University Success Standards, for ELA and Literacy

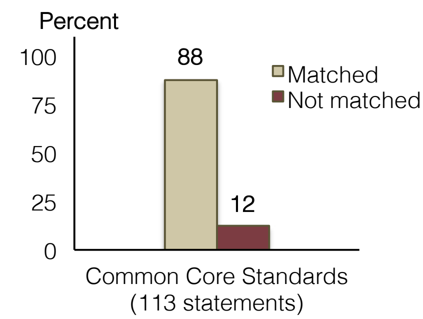


Figure C20. Percentage of Common Core Standards that Match Knowledge and Skills for University Success Standards, by ELA and Literacy Strand

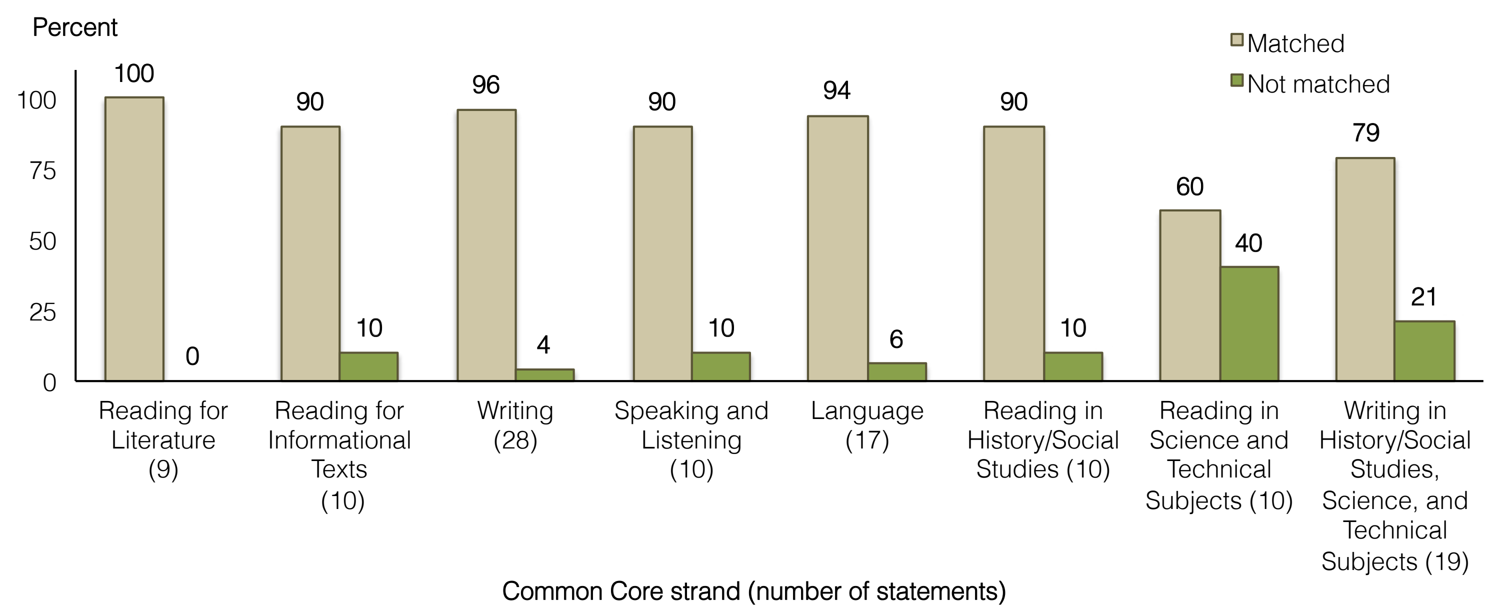


Table C7. Summary for Alignment Between the Knowledge and Skills for University Success Standards and the Common Core Standards in English Language Arts and Literacy

Common Core ELA and literacy strand	Categorical Concurrence	Coverage
Reading for Literature	●	●
Reading for Informational Texts	●	●
Writing	●	●
Speaking and Listening	●	●
Language	●	●
Reading in History/Social Studies	●	●
Reading in Science and Technical Subjects	●	●
Writing in History/Social Studies, Science, and Technical Subjects	●	●

Meets the criterion for Categorical Concurrence (averaged across raters, a single correspondence exists between the two sets of standards);
Does not meet the criterion for Categorical Concurrence;
means that in a majority of topics in the Common Core category have at least one matched comparison standard;
means that in more than one topic in the Common Core category have at least one matched comparison standard;
means that in one or less than one topic in the Common Core category have at least one matched comparison standard.

Breadth of Coverage

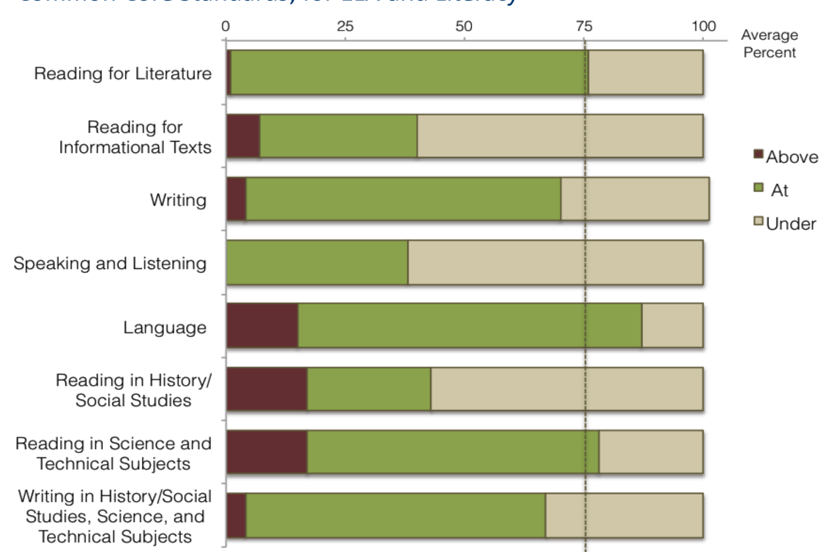
Breadth of Coverage is rated as strong for all the ELA and literacy strands (the majority of Common Core topics have matching comparison standards within them). This means that the KSUS standards that match Common Core standards are distributed across content topics within each Common Core strand. Table C7 presents the alignment summary for the Webb and Cook alignment statistics (Cook, 2005; Cook & Wilmes, 2007; Webb, 2002a).

Depth of Knowledge

Across all matched standards for KSUS in ELA and literacy, an average of 63% (*SD* = 45) fall at the same DOK level as the Common Core standards, with an average of 31% (*SD* = 44) rated lower in cognitive demand and an average of 6% (*SD* = 23) rated higher. Three strands (Reading Standards for Literature, Language Standards, and Reading Standards for Literacy in Science and Technical Subjects) meet the DOK Consistency criterion.

Three strands (Reading Standards for Literature; Language Standards; and Reading Standards for Literacy in Science and Technical Subjects) meet the DOK Consistency criterion, which is based on at least 75% of the matching KSUS standards being at or above the DOK of the corresponding Common Core standard. As Figure C21 shows, some strands fall close to 75% and results would be different if an alternative criterion had been selected.

Figure C21. Percent of Knowledge and Skills for University Success Standards Under, At, or Above the Depth of Knowledge Level of the Common Core Standards, for ELA and Literacy



Note: Total percent may equal slightly above or below 100 due to rounding.

Mathematics Findings for the KSUS Standards

Match

Figure C22 shows the percent of Common Core statements (n = 192) that any of the seven mathematics raters matched to one or more of the KSUS standards (n = 83). The figure collapses findings across the five conceptual categories. In Figure C23, we divide findings by conceptual category. As the bar graphs indicate, the KSUS have matches to 82–89% of the Common Core standards in the Algebra, Functions, and Geometry conceptual categories. They have matches to 53% of the Common Core standards in the Number and Quantity conceptual category and 31% in the Statistics and Probability category.[6] Every conceptual category meets the criterion for the Categorical Concurrence statistic.

[6]The data presented here is based on at least one rater matching at least one standard, however the number of matches for any given standard varies widely. Chapter 4 provides the mean and standard deviation for the number of comparison standards matching Common Core standards in each conceptual category. The Technical Supplement displays the exact standards from the Common Core and the comparison sets that were matched.

Figure C22. Percentage of Common Core Standards that Match Knowledge and Skills for University Success Standards, for Mathematics

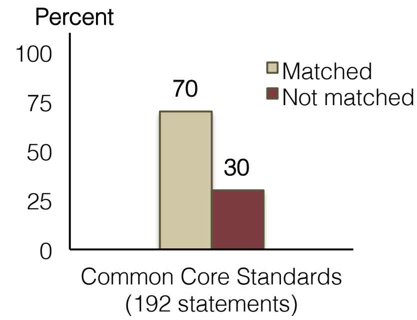


Figure C23. Percentage of Common Core Standards that Match Knowledge and Skills for University Success Standards, by Mathematics Conceptual Category

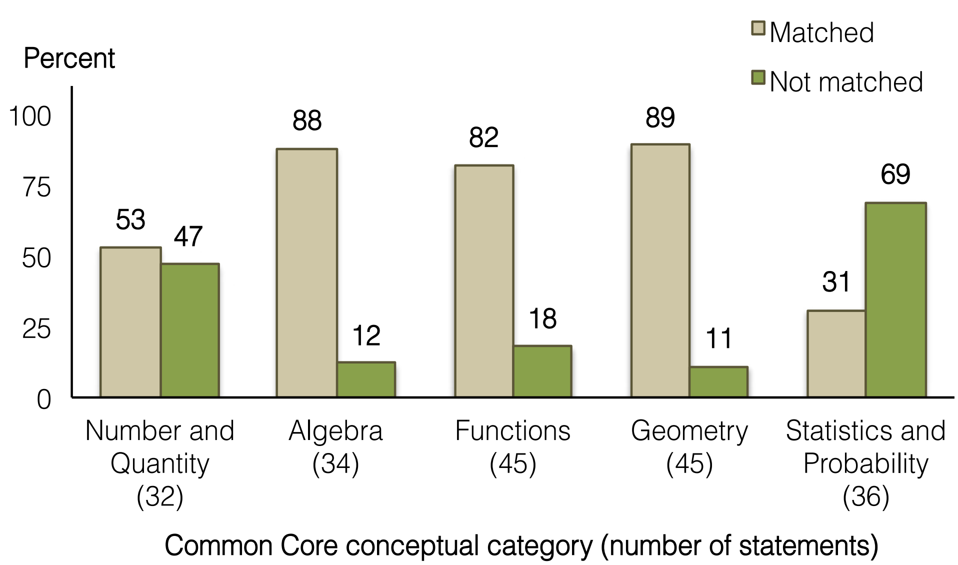


Table C8. Summary for Alignment Between the Knowledge and Skills for University Success Standards and Common Core Standards in Mathematics

Common Core mathematics conceptual category	Categorical Concurrence	Coverage
Number and Quantity		
Algebra		
Functions		
Geometry		
Statistics and Probability		

Meets the criterion for Categorical Concurrence (averaged across raters, a single correspondence exists between the two sets of standards);
Does not meet the criterion for Categorical Concurrence;
means that in a majority of topics in the Common Core category have at least one matched comparison standard; means that in more than one topic in the Common Core category have at least one matched comparison standard; means that in one or less than one topic in the Common Core category have at least one matched comparison standard.

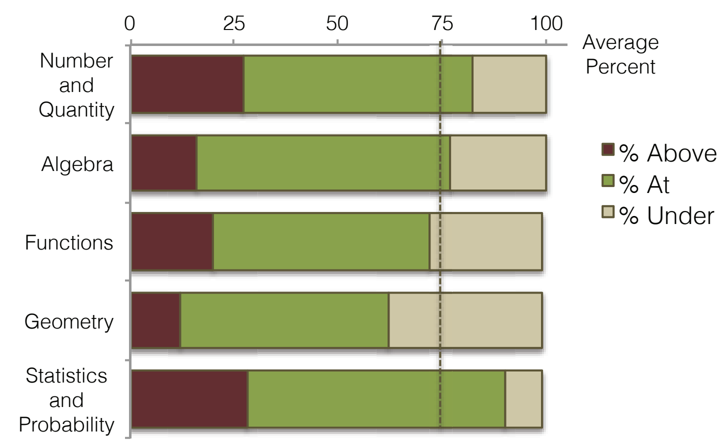
Breadth of Coverage

Breadth of Coverage is strong for all the mathematics conceptual categories (the majority of Common Core clusters have matching comparison standards within them). Table C8 presents the summarized results for mathematics.

Depth of Knowledge

Across conceptual categories in mathematics, for those KSUS standards that match Common Core standards, an average of 55% (*SD* = 47) fall at the same level, an average of 26% (*SD* = 43) are lower, and an average of 18% (*SD* = 37) are higher in cognitive demand. Three of the five Common Core conceptual categories meet the DOK Consistency criterion (see Figure C24), with Functions and Geometry being the two categories that do not meet the criterion. As Figure C24 shows, some categories fall close to 75% and results would be different if an alternative criterion had been selected.

Figure C24. Percent of Knowledge and Skills for University Success Standards Under, At, or Above the Depth of Knowledge Level of the Common Core Standards, for Mathematics

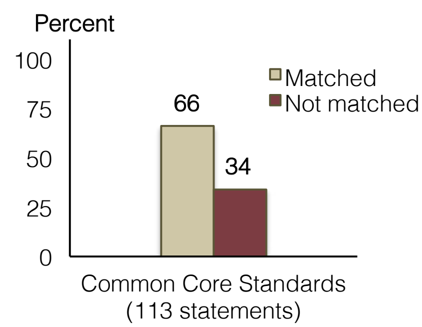


Note: Total percent may equal slightly above or below 100 due to rounding.

International Baccalaureate Standards

We compared the International Baccalaureate (IB) Standards for the Diploma Programme to the Common Core standards. For English language arts we used the course standards for A1 Language, Extended Essay, and Theory of Knowledge. For mathematics we used Mathematical Studies, Standard Level, Higher Level course standards. The study did not include any content from IB “options.” These are additional required components that each IB school selects. The number of options varies by course and by subject. The IB options do not have specific standards.

Figure C25. Percentage of Common Core Standards that Match International Baccalaureate Standards, for ELA and Literacy



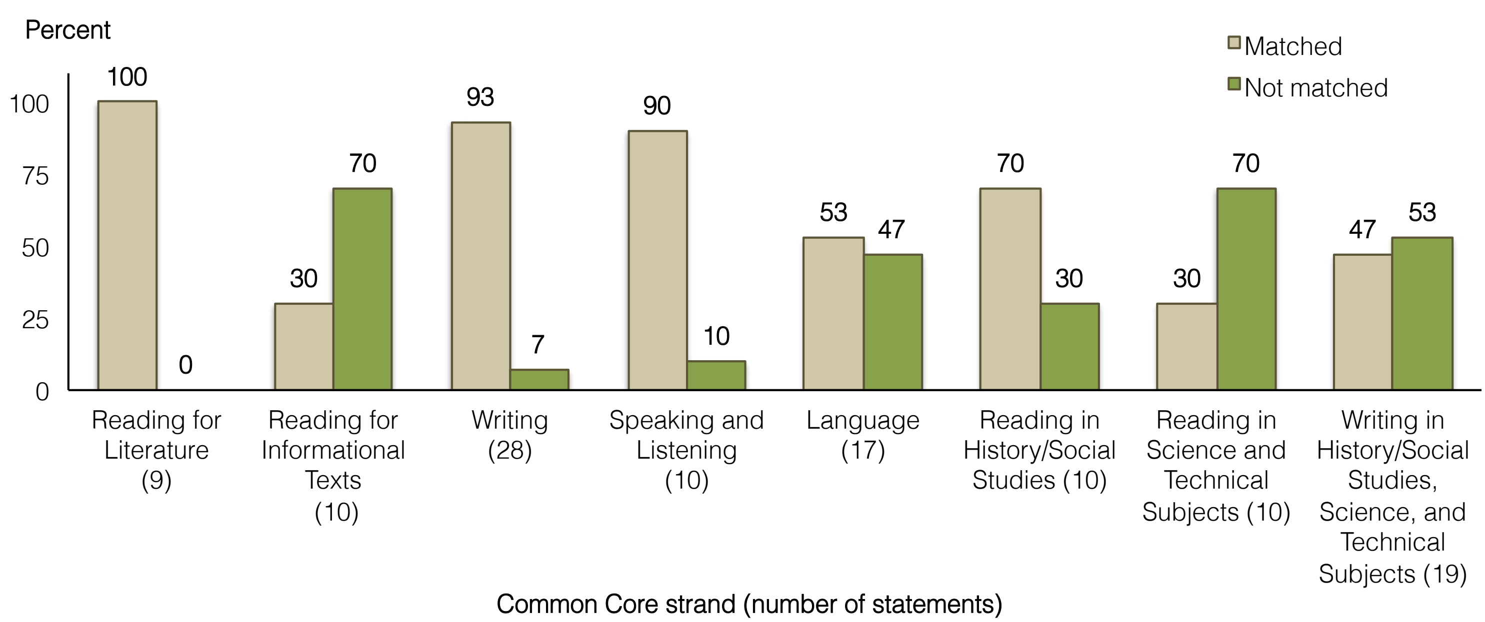
English Language Arts and Literacy Findings for the IB Standards

Match

Recall that raters identified up to three Common Core standards that correspond with each IB standard. Figure C25 shows the percent of Common Core statements (n = 113) that any of the nine English Language Arts (ELA) raters matched to one or more of the IB standards (n = 49). The figure collapses findings across all eight ELA and literacy strands. In Figure C26, we divide findings by strand. As the bar graphs indicate, the IB standards have matches for 100% of the Common Core standards in the Reading for Literature strand. The IB standards have matches to 70–93% of the Common Core standards in the Writing, Speaking and Listening, and Reading Standards in History/Social Studies strands. They have matches to 30–53% of the Common Core standards in the remaining four strands.[7] Six of the eight strands meet the criterion for the Categorical Concurrence statistic; the Language strand and the Reading Standards in Science and Technical Subjects strand do not.

[7]The data presented here is based on at least one rater matching at least one standard, however the number of matches for any given standard varies widely. Chapter 3 provides the mean and standard deviation for the number of comparison standards matching Common Core standards in each strand. The Technical Supplement displays the exact standards from the Common Core and the comparison sets that were matched.

Figure C26. Percentage of Common Core Standards that Match International Baccalaureate Standards, by ELA and Literacy Strand



Depth of Knowledge

Overall, for those IB ELA and literacy standards that match the Common Core, an average of 63% (*SD* = 44) of the IB standards fall at the same Depth of Knowledge (DOK) as the Common Core standards across all strands. An average of 20% (*SD* = 37) of the IB standards are rated lower, and an average of 17% (*SD* = 35) are rated higher in cognitive demand. Of the six strands that meet the criterion for the Categorical Concurrence statistic, four strands meet the criterion for the DOK Consistency statistic. Figure C27 depicts the results graphically. As the figure shows, some strands fall close to 75% and results would therefore be different if an alternative criterion had been selected.

Table C9. Summary for Alignment Between the International Baccalaureate Standards and Common Core Standards in English Language Arts and Literacy

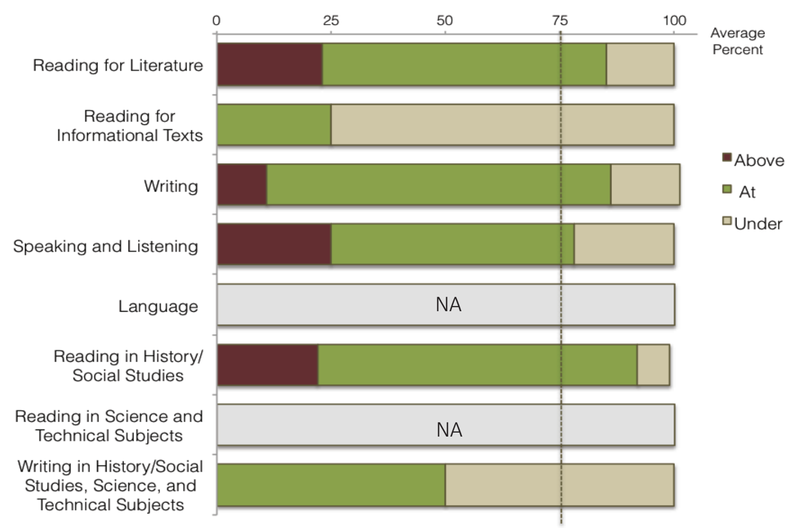
Common Core ELA and literacy strand	Categorical Concurrence	Coverage
Reading for Literature	🟢	●
Reading for Informational Texts	🟢	◐
Writing	🟢	●
Speaking and Listening	🟢	●
Language	🔴	○
Reading in History/Social Studies	🟢	●
Reading in Science and Technical Subjects	🔴	●
Writing in History/Social Studies, Science, and Technical Subjects	🟢	●

🟢 Meets the criterion for Categorical Concurrence (averaged across raters, a single correspondence exists between the two sets of standards);
🔴 Does not meet the criterion for Categorical Concurrence;
● means that in a majority of topics in the Common Core category have at least one matched comparison standard;
◐ means that in more than one topic in the Common Core category have at least one matched comparison standard;
○ means that in one or less than one topic in the Common Core category have at least one matched comparison standard.

Breadth of Coverage

Breadth of Coverage is strong for five of the ELA and literacy strands (for these, the majority of Common Core topics have matching comparison standards within them). This means that the IB standards that match to Common Core standards are distributed across content topics within those five Common Core strands. One strand (Reading for Informational Texts) has moderate coverage. This means that Common Core standards match to IB standards in more than one topic but not a majority of topics within the strand. One strand (Language) has limited coverage; there is one topic that has Common Core standards with matches to the IB standards. Table C9 presents the summary of the alignment statistics for the Webb and Cook alignment statistics (Cook, 2005; Cook & Wilmes, 2007; Webb, 2002).

Figure C27. Percent of International Baccalaureate Standards Under, At, or Above the Depth of Knowledge Level of the Common Core Standards, for ELA and Literacy



Note: Total percent may equal slightly above or below 100 due to rounding.

Mathematics Findings for the IB Standards

Match

Figure C28 shows the percent of Common Core statements (n = 192) that any of the seven mathematics raters matched to one or more of the IB standards (n = 189). The figure collapses findings across the five conceptual categories. In Figure C29, we divide findings by conceptual category. As the bar graphs indicate, the IB standards have matches to 76–97% of the Common Core standards in four of the conceptual categories. They have matches to 44% of the Common Core standards in the Geometry conceptual category.[8] Every conceptual category meets the criterion for the Categorical Concurrence statistic.

[8]The data presented here is based on at least one rater matching at least one standard, however the number of matches for any given standard varies widely. Chapter 4 provides the mean and standard deviation for the number of comparison standards matching Common Core standards in each conceptual category. The Technical Supplement displays the exact standards from the Common Core and the comparison sets that were matched.

Figure C28. Percentage of Common Core Standards that Match International Baccalaureate Standards, for Mathematics

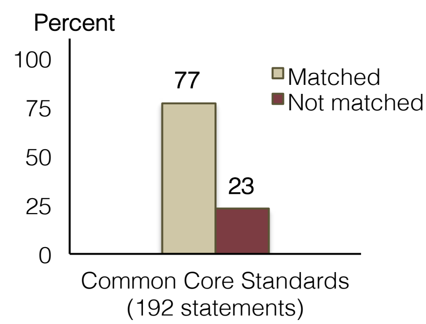


Figure C29. Percentage of Common Core Standards that Match International Baccalaureate Standards, by Mathematics Conceptual Category

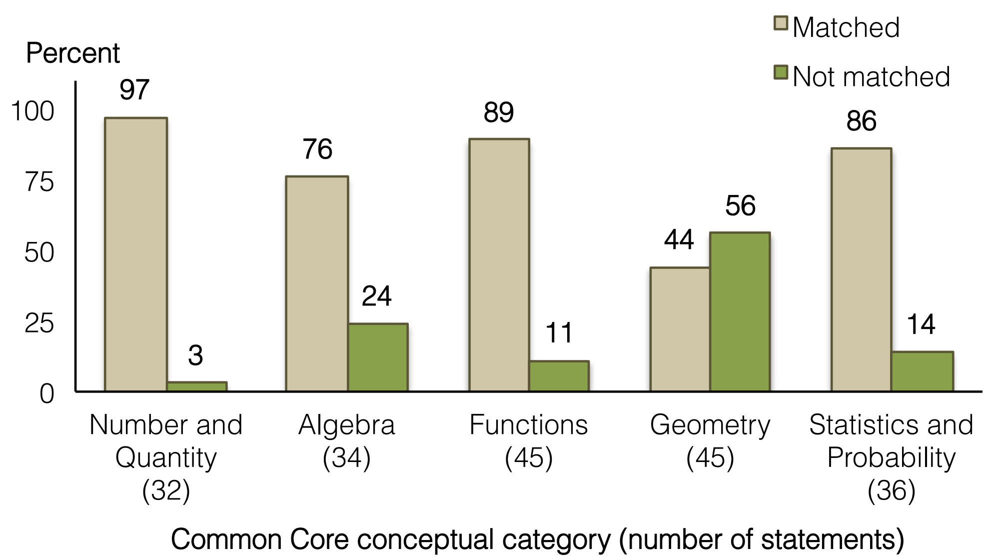


Table C10. Summary of Alignment for the International Baccalaureate Standards and Common Core Standards in Mathematics

Common Core mathematics conceptual category	Categorical Concurrence	Coverage
Number and Quantity	●	●
Algebra	●	●
Functions	●	●
Geometry	●	●
Statistics and Probability	●	●

● Meets the criterion for Categorical Concurrence (averaged across raters, a single correspondence exists between the two sets of standards);
● Does not meet the criterion for Categorical Concurrence;
● means that in a majority of topics in the Common Core category have at least one matched comparison standard;
◐ means that in more than one topic in the Common Core category have at least one matched comparison standard;
○ means that in one or less than one topic in the Common Core category have at least one matched comparison standard.

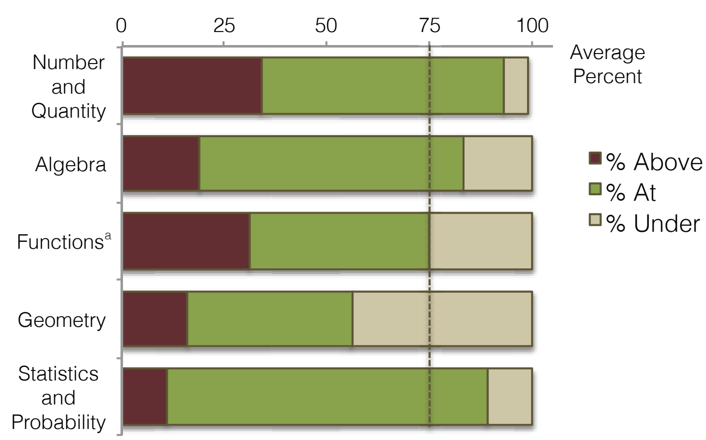
Breadth of Coverage

Breadth of Coverage is strong for all the mathematics conceptual categories (the majority of Common Core clusters have matching comparison standards within them). Table C10 presents the summarized alignment results for mathematics.

Depth of Knowledge

For those IB mathematics standards that match Common Core content, an average of 57% (*SD* = 46) of standards fall at the same level of cognitive demand as the Common Core standards across all mathematics conceptual categories. An average of 19% (*SD* = 38) of the IB standards fall lower in cognitive demand than the Common Core standards, and an average of 23% (*SD* = 40) fall above the Common Core standards. Three of the five Common Core conceptual categories meet the DOK Consistency criterion (see Figure 30), with Functions and Geometry being the two categories that do not meet the criterion. As Figure C30 shows, some categories fall close to or at 75% and results would be different if an alternative criterion had been selected.

Figure C30. Percent of International Baccalaureate Standards Under, At, or Above the Depth of Knowledge Level of the Common Core Standards, for Mathematics



Note: Total percent may equal slightly above or below 100 due to rounding.
[a]The Functions conceptual category does not meet the DOK criterion, but due to rounding, the value appears at exactly 75%.

Number of Alignment Matches for Common Core English Language Arts and Literacy Topics, by Comparison Standard Set

The tables that follow present more information about the distribution of standard matches within each of the eight English language arts (ELA) and literacy strands. Specifically, the tables show the total number of matches that all raters made for each topic (the subareas within a strand). The tables illustrate that there is certain content within a strand that tends to have more concentration of matches than other categories within the same strand. Note that the number of rated statements (shown in parentheses) also varies across topic. Chapter 3 presented the average number of matches per standard for each topic across all comparison sets.

Table D1. Number of Matches Within Each Common Core Topic in the Reading for Literature Strand

Topic (number of statements)	Cumulative number of matches[a]					
	CA	MA	TCCRS	KSUS	IB	**All standards**
Key Ideas and Details (3)	40	20	18	30	22	**130**
Craft and Structure (3)	48	40	33	27	15	**163**
Integration of Knowledge and Ideas (2)	42	22	27	26	15	**132**
Range of Reading and Level of Text Complexity (1)	11	8	12	14	19	**64**

CA = California; MA = Massachusetts; TCCRS = Texas College and Career Readiness Standards; KSUS = Knowledge and Skills for University Success; IB = International Baccalaureate

[a]These matches are cumulative across all raters, meaning that the results show all ratings, including when raters matched the same Common Core standard multiple times within one comparison set of standards and across multiple comparison sets of standards.

Table D2. Number of Matches Within Each Common Core Topic in the Reading for Informational Texts Strand

Topic (number of statements)	Cumulative number of matches[a]					
	CA	MA	TCCRS	KSUS	IB	**All standards**
Key Ideas and Details (3)	12	9	31	12	1	**65**
Craft and Structure (3)	22	30	31	5	0	**88**
Integration of Knowledge and Ideas (3)	18	6	24	4	4	**56**
Range of Reading and Level of Text Complexity (1)	2	0	3	3	5	**13**

CA = California; MA = Massachusetts; TCCRS = Texas College and Career Readiness Standards; KSUS = Knowledge and Skills for University Success; IB = International Baccalaureate

[a]These matches are cumulative across all raters, meaning that the results show all ratings, including when raters matched the same Common Core standard multiple times within one comparison set of standards and across multiple comparison sets of standards.

Table D3. Number of Matches Within Each Common Core Topic in the Writing Strand

Topic (number of statements)	Cumulative number of matches[a]					
	CA	MA	TCCRS	KSUS	IB	**All standards**
Text Types and Purposes (19)	125	44	46	92	76	**383**
Production and Distribution of Writing (3)	33	20	61	31	17	**162**
Research to Build and Present Knowledge (5)	21	10	78	44	55	**208**
Range of Writing (1)	5	3	11	3	6	**28**

CA = California; MA = Massachusetts; TCCRS = Texas College and Career Readiness Standards; KSUS = Knowledge and Skills for University Success; IB = International Baccalaureate

[a]These matches are cumulative across all raters, meaning that the results show all ratings, including when raters matched the same Common Core standard multiple times within one comparison set of standards and across multiple comparison sets of standards.

Table D4. Number of Matches Within Each Common Core Topic in the Speaking and Listening Strand

Topic (number of statements)	Cumulative number of matches[a]					
	CA	MA	TCCRS	KSUS	IB	**All standards**
Comprehension and Collaboration (7)	54	20	90	19	24	**207**
Presentation of Knowledge and Ideas (3)	98	25	42	8	14	**187**

CA = California; MA = Massachusetts; TCCRS = Texas College and Career Readiness Standards; KSUS = Knowledge and Skills for University Success; IB = International Baccalaureate
[a]These matches are cumulative across all raters, meaning that the results show all ratings, including when raters matched the same Common Core standard multiple times within one comparison set of standards and across multiple comparison sets of standards.

Table D5. Number of Matches Within Each Common Core Topic in the Language Strand

Topic (number of statements)	Cumulative number of matches[a]					
	CA	MA	TCCRS	KSUS	IB	**All standards**
Conventions of Standard English (6)	26	36	18	48	0	**128**
Knowledge of Language (2)	5	6	4	12	0	**27**
Vocabulary Acquisition and Use (9)	13	32	55	40	2	**142**

CA = California; MA = Massachusetts; TCCRS = Texas College and Career Readiness Standards; KSUS = Knowledge and Skills for University Success; IB = International Baccalaureate
[a]These matches are cumulative across all raters, meaning that the results show all ratings, including when raters matched the same Common Core standard multiple times within one comparison set of standards and across multiple comparison sets of standards.

Table D6. Number of Matches Within Each Common Core Topic in the Reading for Literacy in History/ Social Studies Strand

Topic (number of statements)	Cumulative number of matches[a]					
	CA	MA	TCCRS	KSUS	IB	**All standards**
Key Ideas and Details (3)	20	1	13	5	2	**41**
Craft and Structure (3)	19	3	6	6	1	**35**
Integration of Knowledge and Ideas (3)	16	1	12	7	9	**45**
Range of Reading and Level of Text Complexity (1)	1	0	1	3	1	**6**

CA = California; MA = Massachusetts; TCCRS = Texas College and Career Readiness Standards; KSUS = Knowledge and Skills for University Success; IB = International Baccalaureate
[a]These matches are cumulative across all raters, meaning that the results show all ratings, including when raters matched the same Common Core standard multiple times within one comparison set of standards and across multiple comparison sets of standards.

Table D7. Number of Matches Within Each Common Core Topic in the Reading for Literacy in Science and Technical Subjects Strand

Topic (number of statements)	Cumulative number of matches[a]					
	CA	MA	TCCRS	KSUS	IB	**All standards**
Key Ideas and Details (3)	3	1	10	3	2	**19**
Craft and Structure (3)	5	1	14	5	0	**25**
Integration of Knowledge and Ideas (3)	1	0	16	1	3	**21**
Range of Reading and Level of Text Complexity (1)	0	0	2	0	1	**3**

CA = California; MA = Massachusetts; TCCRS = Texas College and Career Readiness Standards; KSUS = Knowledge and Skills for University Success; IB = International Baccalaureate
[a]These matches are cumulative across all raters, meaning that the results show all ratings, including when raters matched the same Common Core standard multiple times within one comparison set of standards and across multiple comparison sets of standards.

Table D8. Number of Matches Within Each Common Core Topic in the Writing for Literacy in History/ Social Studies, Science, and Technical Subjects Strand

Topic (number of statements)	Cumulative number of matches[a]					
	CA	MA	TCCRS	KSUS	IB	**All standards**
Text Types and Purposes (12)	13	2	19	22	4	**60**
Production and Distribution of Writing (3)	4	5	16	12	2	**39**
Research to Build and Present Knowledge (3)	3	2	16	18	11	**50**
Range of Writing (1)	0	0	4	1	2	**7**

CA = California; MA = Massachusetts; TCCRS = Texas College and Career Readiness Standards; KSUS = Knowledge and Skills for University Success; IB = International Baccalaureate
[a]These matches are cumulative across all raters, meaning that the results show all ratings, including when raters matched the same Common Core standard multiple times within one comparison set of standards and across multiple comparison sets of standards.

Number of Alignment Matches for Common Core Clusters, by Comparison Standard Set for Mathematics

The tables that follow present more information about the distribution of standard matches within each of the five Common Core mathematics conceptual categories. Specifically, the tables show the total number of matches that all raters made for each cluster (the subareas within a conceptual category). The tables illustrate that there are certain clusters within a category that tend to have more concentration of matches than other clusters within the same category. Note that the number of rated statements (shown in parentheses) also varies across cluster. Chapter 4 presented the average number of matches per standard for each cluster across all comparison sets.

Table D9. Number of Matches Within Each Common Core Cluster in the Number and Quantity Conceptual Category

Cluster (number of statements)	Cumulative number of matches[a]					
	CA	MA	TCCRS	KSUS	IB	**All standards**
Extend the properties of exponents to rational exponents (2)	15	11	4	19	1	**50**
Use properties of rational and irrational numbers (1)	7	4	1	0	2	**14**
Reason quantitatively and use units to solve problems (3)	1	18	22	8	9	**58**
Perform arithmetic operations with complex numbers (3)	13	17	19	4	5	**58**
Represent complex numbers and their operations on the complex plane (3)	23	19	2	2	9	**55**
Use complex numbers in polynomial identities and equations (3)	4	10	0	0	13	**27**
Represent and model with vector quantities (3)	6	7	0	1	23	**37**
Perform operations on vectors (7)	19	22	0	1	24	**66**
Perform operations on matrices and use matrices in applications (7)	23	9	0	0	36	**68**

CA = California; MA = Massachusetts; TCCRS = Texas College and Career Readiness Standards; KSUS = Knowledge and Skills for University Success; IB = International Baccalaureate

[a]These matches are cumulative across all raters, meaning that the results show all ratings, including when raters matched the same Common Core standard multiple times within one comparison set of standards and across multiple comparison sets of standards.

Table D10. Number of Matches Within Each Common Core Cluster in the Algebra Conceptual Category

Cluster (number of statements)	Cumulative number of matches[a]					
	CA	MA	TCCRS	KSUS	IB	**All standards**
Interpret the structure of expressions (4)	8	5	16	19	8	**56**
Write expressions in equivalent forms to solve problems (5)	30	20	13	31	36	**130**
Perform arithmetic operations on polynomials(1)	12	6	6	2	1	**27**
Understand the relationship between zeros and factors of polynomials (2)	6	4	0	5	16	**31**
Use polynomial identities to solve problems (2)	7	10	0	0	6	**23**
Rewrite rational expressions (2)	25	5	6	13	2	**51**
Create equations that describe numbers or relationships (4)	8	9	35	12	5	**69**
Understand solving equations as a process of reasoning and explain the reasoning (2)	22	10	9	9	2	**52**
Solve equations and inequalities in one variable (4)	40	44	6	20	8	**118**
Solve systems of equations (5)	53	32	15	8	21	**129**
Represent and solve equations and inequalities graphically (3)	16	17	19	14	14	**80**

CA = California; MA = Massachusetts; TCCRS = Texas College and Career Readiness Standards; KSUS = Knowledge and Skills for University Success; IB = International Baccalaureate
[a]These matches are cumulative across all raters, meaning that the results show all ratings, including when raters matched the same Common Core standard multiple times within one comparison set of standards and across multiple comparison sets of standards.

Table D11. Number of Matches Within Each Common Core Cluster in the Functions Conceptual Category

Cluster (number of statements)	Cumulative number of matches[a]					
	CA	MA	TCCRS	KSUS	IB	**All standards**
Understand the concept of a function and use function notation (3)	16	22	23	18	12	**91**
Interpret functions that arise in applications in terms of the context (3)	16	23	17	11	18	**85**
Analyze functions using different representations (10)	38	30	39	20	86	**213**
Build a function that models a relationship between two quantities (5)	16	34	24	13	25	**112**
Build new functions from existing functions (7)	18	21	12	14	22	**87**
Construct and compare linear and exponential models and solve problems (7)	20	31	30	23	28	**132**
Interpret expressions for functions in terms of the situation they model (1)	0	1	1	3	2	**7**
Extend the domain of trigonometric functions using the unit circle (4)	20	14	1	10	32	**77**
Model periodic phenomena with trigonometric functions (3)	17	3	1	6	35	**62**
Prove and apply trigonometric identities (2)	20	15	0	4	12	**51**

CA = California; MA = Massachusetts; TCCRS = Texas College and Career Readiness Standards; KSUS = Knowledge and Skills for University Success; IB = International Baccalaureate
[a]These matches are cumulative across all raters, meaning that the results show all ratings, including when raters matched the same Common Core standard multiple times within one comparison set of standards and across multiple comparison sets of standards.

Table D12. Number of Matches Within Each Common Core Cluster in the Geometry Conceptual Category

Cluster (number of statements)	Cumulative number of matches[a]					
	CA	MA	TCCRS	KSUS	IB	**All standards**
Experiment with transformations in the plane (5)	7	10	18	2	3	**40**
Understand congruence in terms of rigid motions (3)	12	5	9	1	0	**27**
Prove geometric theorems (3)	11	6	8	16	0	**41**
Make geometric constructions (2)	7	8	5	5	1	**26**
Understand similarity in terms of similarity transformations (5)	3	5	6	3	0	**17**
Prove theorems involving similarity (2)	16	18	6	9	0	**49**
Define trigonometric ratios and solve problems involving right triangles (3)	29	25	16	11	25	**106**
Apply trigonometry to general triangles (3)	12	17	1	5	15	**50**
Understand and apply theorems about circles (4)	15	16	4	4	1	**40**
Find arc lengths and areas of sectors of circles (1)	1	1	2	0	4	**8**
Translate between the geometric description and the equation for a conic section (3)	24	11	2	14	0	**51**
Use coordinates to prove simple geometric theorems algebraically (4)	24	32	20	14	16	**106**
Explain volume formulas and use them to solve problems (3)	21	6	12	6	11	**56**
Visualize relationships between two-dimensional and three-dimensional objects (1)	1	4	5	4	3	**17**
Apply geometric concepts in modeling situations (3)	1	4	18	10	5	**38**

CA = California; MA = Massachusetts; TCCRS = Texas College and Career Readiness Standards; KSUS = Knowledge and Skills for University Success; IB = International Baccalaureate
[a]These matches are cumulative across all raters, meaning that the results show all ratings, including when raters matched the same Common Core standard multiple times within one comparison set of standards and across multiple comparison sets of standards.

Table D13. Number of Matches Within Each Common Core Cluster in the Statistics and Probability Conceptual Category

Cluster (number of statements)	Cumulative number of matches[a]					
	CA	MA	TCCRS	KSUS	IB	**All standards**
Summarize, represent, and interpret data on a single count or measurement variable (4)	34	36	52	13	54	**189**
Summarize, represent, and interpret data on two categorical and quantitative variables (5)	19	32	58	16	32	**157**
Interpret linear models (3)	10	7	5	1	18	**41**
Understand and evaluate random processes underlying statistical experiments (2)	10	21	23	0	6	**60**
Make inferences and justify conclusions from sample surveys, experiments, and observational studies (4)	3	18	43	1	3	**68**
Understand independence and conditional probability and use them to interpret data (5)	26	1	23	1	41	**92**
Use the rules of probability to compute probabilities of compound events in a uniform probability model (4)	28	7	7	0	26	**68**
Calculate expected values and use them to solve problems (4)	16	0	3	0	24	**43**
Use probability to evaluate outcomes of decisions (5)	2	0	9	0	12	**23**

CA = California; MA = Massachusetts; TCCRS = Texas College and Career Readiness Standards; KSUS = Knowledge and Skills for University Success; IB = International Baccalaureate
[a]These matches are cumulative across all raters, meaning that the results show all ratings, including when raters matched the same Common Core standard multiple times within one comparison set of standards and across multiple comparison sets of standards.